Positioning Based on 2-D Angle of Arrival Estimation

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Abstract—We study the problem of determining the position of a node via the exploitation of 2-D angle of arrival estimation (azimuth and elevation), using orthogonal linear arrays. We also consider the presence of hardware impairments that include RF carrier frequency, phase, and DC offsets. We show that the algorithm is robust to the hardware impairments and it provides good performance for a wide range of distances from the receiver. Both the Cramer-Rao bound and simulation results are reported and they are compared with the root-MUSIC performance.

I. INTRODUCTION

Recently, the topic of positioning in wireless networks has drawn considerable attention. In vehicular networks, the knowledge of the nodes position can be useful for several ITS (Intelligent Transportation System) applications, e.g., intersection safety, accident warning. In this case, the use of a roadside unit in a line-of-sight (LOS) situation becomes needful to establish a V2I (Vehicular-to-Infrastructure) communication channel capable of estimate the vehicle position and communicate it to the other nodes. As it is well known, the position of a node can be determined by a variety of ways, such as Time of Arrival (ToA), Time Difference of Arrival (TDoA), or Angle of Arrival (AoA). Neglecting the first two techniques that require time synchronization and the capability of high sampling rate, there exist several algorithms that allow us to estimate the AoA. The best performance can be probably obtained with the subspace-based methods [2]-[3], but these require large computational resources that are not always available. Also, the hardware impairments, always present in reality, require other computational efforts to ensure the achievement of robust performance. In this paper we present a positioning system based on the 2-D extension of the AoA estimator in [4], exploiting L-shaped arrays [5] at known height. We assume herein a planewave LOS propagation scenario, due to the presence of an overplaced road-side unit. The 2-D AoA estimator allows the compensation of the carrier frequency offset (in which we can include the Doppler effect), and of the DC offset (a nuisance problem for the RF direct-conversion receiver). To the best of our knowledge, these impairments have not been considered before in this field. Furthermore, pre-calibration of the array can be done with a known position source.

The system model has been derived by the experimental characterization of a hardware testbed, equipped with a linearly equispaced array, four RF direct-conversion receivers, and an eight channels acquisition board, with FPGA.

II. SYSTEM MODEL DESCRIPTION

We consider a node that emits a single tone signal $s(t) = e^{j2\pi f_0 t}$ with frequency $f_0$, and a receiver equipped with a 3D L-shaped linearly equispaced array placed at a height $z_0$ and arranged as shown in Fig. 1. The 3D L-shaped array [5] comprises three orthogonal linearly equispaced antenna arrays which we refer to as subarray. This array configuration uses a subarray in each axis and it has a total of $3M - 2$ elements.

Assuming a planewave LOS propagation scenario, the incident signals at the $i$-th antenna element (sensor) of the $a$-th subarray can be written as

$$x_{RF}^{(a,i)}(t) = \rho e^{j2\pi f_0 t - \tau^{(a)}} - \Delta t^{(a,i)} + w_{RF}^{(a,i)}(t)$$

where $\tau^{(a)}$ is the propagation delay between the emitter and the first element of each subarray, $\rho$ is the propagation loss that we assume to be time invariant during the 2-D AoA estimation, and $w_{RF}^{(a,i)}(t)$ is the additive noise. It should be noted that $i \in \{1, \ldots, M\}$, while $a \in \{x, y, z\}$. Now, let us assume that the planewave impinges on the sensors of every subarray with azimuth $\phi$ and elevation $\vartheta$. Then, the differential propagation delay between the first sensor and the $i$-th sensor of a given subarray $a$ can be written as

$$\Delta t^{(a,i)} = \begin{cases} 
\frac{d}{c_0} (i - 1) \cos(\phi) \sin(\vartheta), & a = x \\
\frac{d}{c_0} (M - i) \sin(\phi) \sin(\vartheta), & a = y \\
\frac{d}{c_0} (i - 1) \cos(\vartheta), & a = z
\end{cases}$$

where $d$ is the interelement distance, and $c_0$ is the speed of
light. It should be noted that \( \phi \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \) and \( \vartheta \in \left[ 0, \frac{\pi}{2} \right] \), in order to identify the source position in the half plane \( x \geq 0 \), with \( z_0 > 0 \). However, it is also possible to identify the source position in the whole plane \( z = 0 \) using the extended arctangent function, \text{arctan}2.

The received RF signals are down converted to low frequency \( f_0 \) using a direct-conversion receiver architecture, with a local oscillator frequency \( f_{LO}^{(a,i)} = f_0 - f_0 - \Delta f^{(a,i)} \). As it will be explained in the following, we choose \( f_0 \) to be larger than zero to allow the compensation of DC offset components. Furthermore, we assume that the local oscillator of subarray \( a \) and element \( i \) is affected by a frequency offset \( \Delta f^{(a,i)} \) with respect to the transmitter carrier frequency.

Now, assuming to sample the signals with period \( T \), the sequence of complex samples associated to the \( i \)-th element of the \( a \)-th subarray, can be written as

\[
x_n^{(a,i)} = \alpha e^{j\psi^{(a,i)} - \Phi^{(a,i)}} + w_n^{(a,i)} + w_{DC,n}^{(a,i)}
\]

where

\[
\psi^{(a,i)} = 2\pi(f_0 + \Delta f^{(a,i)}) nT - 2\pi f_{0,RF} \vartheta^{(a,i)} - 2\pi f_{0,RF} \Delta f^{(a,i)},
\]

and \( \alpha \) is the time invariant channel gain that includes the propagation loss \( \rho \) and the receiver gain. Furthermore, the phase \( \Phi^{(a,i)} \) is introduced by non co-phasal RF oscillators and by an antenna array that is not perfectly calibrated, i.e., an antenna array that does not have equalized delays among the receiver paths.

Besides the carrier frequency and phase offsets, the hardware may exhibit mutual coupling between internal and external signals that introduce a DC offset component \( w_{DC,n}^{(a,i)} \). We model it as circularly symmetric complex Gaussian with mean \( m_{DC} \) and correlation

\[
t_{DC}^{(a,b,i,j)}(k) = E\{w_{DC,n}^{(a,i)} w_{DC,n+k}^{(b,j)}\} = N_0 \delta_{a,b} \delta_{i,j}
\]

where \( \delta_{i,j} \) is the Kronecker delta. The background noise contribution \( w_n^{(a,i)} \) is also assumed circularly symmetric complex Gaussian, with zero mean, variance \( N_0/2 \) per component, and correlation

\[
\delta^{(a,b,i,j)}(k) = E\{w_n^{(a,i)} w_{n+k}^{(b,j)}\} = N_0 \delta_{a,b} \delta_{i,j} \delta(k)
\]

where \( \delta(k) = 1 \) for \( k = 0 \) and 0 otherwise.

We now define the signal-to-background noise ratio as \( SNR = \frac{\alpha^2}{2\sigma_{DC}} \) and the signal-over-dc offset ratio as \( SDR = \frac{\alpha^2}{2\sigma_{DC}} \) where \( N_0 = \sigma_{DC}^2 + |m_{DC}|^2 \) and \( \sigma_{DC}^2 \) is the variance of the DC offset.

Our goal is the detection of the emitter position through the estimation of the 2-D AoA, obtainable exploiting the phase differences \( 2\pi f_{0,RF} \Delta f^{(a,i)} \) in (4). Then, the knowledge of the relative height from the emitter to the receiver \( z_0 \) allows us to locate the source at the position \((x_0, y_0)\).

### III. IMPAIRMENTS COMPENSATION AND 2-D AOA ESTIMATION

We now describe the proposed algorithm for the estimation of the 2-D AoA in the presence of the hardware impairments.

#### A. Impairments Compensation

To remove the DC offset, we down-convert the RF signals for each antenna element to a low frequency \( f_0 \) and we sample them to obtain (3). Then, we implement a digital band-pass filter with impulse response \( h_n \) to filter the signals \( x_n^{(a,i)} \) and obtain

\[
x_{F,n}^{(a,i)} = \alpha e^{j(\psi^{(a,i)} - \Phi^{(a,i)})} + w_{F,n}^{(a,i)}
\]

where we have assumed the carrier frequency offsets to be small compared to the filter bandwidth. The filtered noise \( w_{F,n}^{(a,i)} \) has mean \( m_{w,F} = m_{DC} A_h \) with \( A_h = \sum_n h_n \), and correlation

\[
t_{F}^{(a,b,i,j)}(k) = (N_0 r_h(k) + N_1 A_r) \delta_{a,b} \delta_{i,j}
\]

where \( r_h(k) \) is the autocorrelation of the impulse response of the filter and \( A_r = A_r^2 \) is the area of the autocorrelation of the impulse response of the filter. With a selective filter, the DC offset can be eliminated. In general the SNR at the filter output can be defined as \( SNR_{x,F} = \frac{x^2}{M \cdot w_{F}} \) with \( m_{w,F} = m_{w,F}^{(a,i)}(0) = N_0 r_h(0) + A_r N_1 \).

Now, to proceed we assume the carrier frequency offsets of the receivers connected to the same subarray to be identical, i.e., \( \Delta f^{(a,i)} = \Delta f^{(a,j)} \), \( \forall i \in \{1, M\} \). Then, to compensate them, we perform a differential operation among pairs of signals from elements that belong to the same subarray, either correlating distinct signal pairs (DPC, Distinct antenna elements Pairs Combining - \( z_{n}^{(a,i)} = z_{n}^{(a,2i-1)} , x_{F,n}^{(a,2i-1)} \)) or correlating all adjacent signal pairs (APC, All antenna elements Pairs Combining - \( z_{n}^{(a,i)} = z_{n}^{(a,i)} , x_{F,n}^{(a,i)} \)). So, the signals \( z_n^{(a,i)} \) at the combiner output can be obtained as

\[
z_n^{(a,i)} = \alpha e^{j(\psi^{(a,i)} - \Phi^{(a,i)})} + \tilde{v}_n^{(a,i)}
\]

where, indicating \( \lambda_0 = \frac{c_0}{f_0,RF} \) and \( \kappa = 2\pi \frac{d}{\lambda_0} \)

\[
\begin{align*}
\tilde{v}_n^{(a)} &= \begin{cases}
\kappa \cos(\phi) \sin(\vartheta) = \kappa \frac{\sin(\vartheta) \sin(\phi)}{\sqrt{x_0^2 + y_0^2 + z_0^2}}, & a = x \\
-\kappa \sin(\phi) \sin(\vartheta) = -\kappa \frac{y_0}{\sqrt{x_0^2 + y_0^2 + z_0^2}}, & a = y \\
\kappa \cos(\vartheta) = \kappa \frac{x_0}{\sqrt{x_0^2 + y_0^2 + z_0^2}}, & a = z \\
\end{cases}
\end{align*}
\]

and \( \alpha = \alpha^2 \), while \( \tilde{\Phi}^{(a,i)} = \Phi^{(a,2i-1)} - \Phi^{(a,2i)} \) or \( \tilde{\Phi}^{(a,i)} = \Phi^{(a,i)} - \Phi^{(a,i+1)} \) for DPC and APC, respectively. Also, the noise process \( \tilde{v}_n^{(a)} \) has statistical power \( M \sigma_{DC} \). It should be noted that \( i \in \{1, \ldots, M/2\} \) for the DPC algorithm, while \( i \in \{1, \ldots, M-1\} \) for the APC.

As (9) reveals, the time variant phase ambiguity introduced by the carrier frequency offset is removed, and we are left with the phase \( \tilde{\psi}^{(a)} \) of interest that is function of the angles of arrival (\( \phi, \vartheta \)). Furthermore, it is easy to prove that the SNR at the combiner output is approximately

\[
SNR_z \approx \frac{SNR_{x,F}}{2}
\]

in the high signal-to-noise ratio region.

Observing (9), we notice that there is still a phase ambiguity \( \tilde{\Phi}^{(a,i)} \) that has to be removed from the useful signal. It can be
removed either using a local reference signal [8] or performing a DoA pre-estimation with a transmitting node at an arbitrarily known position. We consider the latter approach. In particular, we assume the reference node to be positioned such that the angles of arrival have $\phi_{REF} = 90^\circ$, $\vartheta_{REF} = 90^\circ$. Now, during the transmission from the reference node, the combiner output signals in (9) read

$$z_{REF,n}^{(a,i)} = \hat{a} e^{j(\psi^{(a,i)} - \hat{\phi}^{(a,i)})} + \tilde{w}_{REF,n}^{(a,i)}$$

where

$$\tilde{w}_{REF} = \left\{ \begin{array}{ll} 0, & \forall a \in \{ x, z \} \\ -\kappa, & a = y. \end{array} \right.$$  (13)

The signal $w_{REF,n}^{(a,i)}$ is the noise process for the reference transmitter that has statistical power $M_w$. We then proceed by performing another differential operation with a lag of $N$ samples between the reference signals (12) and the unknown AoA signals (9) (because we assume to time multiplex the transmission of the reference signal with that of the unknown AoA signal) to obtain

$$z_n^{(a,i)} = z_{n+N}^{(a,i)} \cdot z_{REF,n}^{(a,i)*} = A e^{j(\psi^{(a,i)} - \psi_{REF}^{(a,i)})} + \tilde{w}_n^{(a,i)}$$  (14)

where the noise term has statistical power $M_w = 2AM_w + (M_w)^2$. As (14) shows, the phase ambiguity $\tilde{\phi}^{(a,i)}$ is removed. Also, in order to simplify the analytical derivation, we have assumed that the reference node is placed at the same distance of the source, yielding the same channel coefficient $\alpha$.

### B. 2-D AoA Estimation

The 2-D AoA can be estimated from (14) performing averaging in time (over $N$ samples) and in space (over $L$ samples, with $L = M/2$ for the DPC and $L = M - 1$ for the APC), as

$$z^{(a)} = \frac{1}{NL} \sum_{n=0}^{N-1} \sum_{i=1}^{L} z_n^{(a,i)} = \left\{ \begin{array}{ll} A e^{j(\psi^{(a)} - \psi_{REF}^{(a)})} + W^{(a)}, & \text{DPC} \\ (A + \gamma_w^{(a)}) e^{j(\tilde{\theta}^{(a)} - \tilde{\psi}_{REF}^{(a)})} + W_t^{(a)}, & \text{APC} \end{array} \right.$$  (15)

where

$$W_t^{(a)} = \frac{2}{NM} \sum_{n=0}^{N-1} \sum_{i=1}^{M/2} \tilde{w}_n^{(a,i)}$$  (16)

is the complex noise contribution that has statistical power $M_{W_t} = \frac{2}{NM} M_w$. It should be noted that we have assumed the differential noise $\tilde{w}_n^{(a,i)}$ to be spatially and temporally uncorrelated. Furthermore, it is easy to prove that Gaussianity holds true for both the real and the imaginary part of $W^{(a)}$. For the APC algorithm, it can be proved that averaging the noise samples $\tilde{w}_n^{(a,i)}$ yields

$$\frac{1}{N(M-1)} \sum_{n=0}^{N-1} \sum_{i=1}^{M-1} \tilde{w}_n^{(a,i)} = \gamma_w^{(a,i)} e^{j(\psi^{(a)} - \psi_{REF}^{(a)})} + W_t^{(a)}$$  (17)

where $\gamma_w^{(a,i)}$ is a real process that contributes to the useful signal and $W_t^{(a)}$ is the complex noise contribution. We omit their derivation for space limitation.

Finally, we can process the signals $z^{(a)}$, $\forall a \in \{ x, y, z \}$ to estimate the angles of arrival as follows

$$\begin{align*}
\tilde{\phi} &= -\arctan\left( \frac{\Delta \hat{\phi}}{\sqrt{\hat{\psi}^2 + \Delta \hat{\psi}^2}} \right) \\
\tilde{\vartheta} &= \arctan\left( \frac{\Delta \hat{\psi}}{\sqrt{\hat{\psi}^2 + \Delta \hat{\psi}^2}} \right)
\end{align*}$$  (18)

where $\Delta$ is the argument operator.

### C. Estimation of the position

If we assume to know the height of the array $z_0$, it is possible to determine the position, i.e., the coordinates $(x_0, y_0)$, of the emitter from the AoA estimates as follows

$$\begin{align*}
x_0 &= x_0 \cos(\tilde{\phi}) \tan(\tilde{\vartheta}) \\
y_0 &= z_0 \sin(\tilde{\phi}) \tan(\tilde{\vartheta})
\end{align*}$$  (19)

It should be noted that the position cannot be determined if the emitter lies in the plane of the array ($z_0 = 0$).

### IV. PERFORMANCE ANALYSIS

In this section we report the performance analysis of the proposed algorithm in terms of root mean squared error RMSE, first studying the Cramer-Rao bound of the error in positioning and then comparing it with the results obtained from simulations. In the numerical examples, we have assumed $M = 4$, $N = 1$, $f_{0,REF} = 5.805$ GHz, $f_0 = 3$ MHz, and antenna elements spaced by $d = \lambda/2$. The carrier frequency offsets have been modeled as spatially independent Gaussian random variables with mean value $m_{DC} = 15$ kHz and standard deviation $\sigma_{\Delta f} = \{0, 10\}$ kHz. The phase offsets $\Phi^{(a,i)}$ have been considered as spatially independent uniform random variables in $[0, 2\pi]$. Finally, the DC offset $\gamma_{DC}^{(a,i)}$ has been drawn for each estimation window from a complex normal distribution, with variance $\sigma_{\Delta DC}^2$ that is the $10\%$ of the mean $m_{DC}$ (as revealed by the testbed characterization).

#### A. Cramer-Rao Bound

The MSE in the coordinates estimation

$$MSE(x_0) = E\{|x_0 - \hat{x}_0|^2\}, MSE(y_0) = E\{|y_0 - \hat{y}_0|^2\}$$  (20)

can be lower bounded with the Cramer-Rao bound [9], [10]. To benchmark the best attainable performance we evaluate the Cramer-Rao bound in the absence of DC offset and under identical carrier frequency offsets for all antenna channels. Although the derivation has been omitted for space limitation, for the DPC it can be shown that

$$\begin{align*}
CRB(x_0) &= \frac{x_0^2 + \hat{\Delta}^2 + \hat{\Delta}^2 + \hat{\Delta}^2 + \hat{\Delta}^2}{2\hat{\psi}_t} \\
CRB(y_0) &= \frac{\hat{\psi}_t^2 + \hat{\Delta}^2 + \gamma_w}{2\hat{\psi}_t^2}
\end{align*}$$  (21)
The ratio $\frac{A^2}{\sigma_w^2}$ can be related to the SNR at the receiver filter output as

$$A^2 = \frac{NM SNR_z^2}{2SNR_z + 1} \approx \frac{NM}{8} SNR_{xf}$$

(22)

that holds true for high SNRs. If the array calibration is not necessary, i.e., it is not necessary to perform the second differential operation to eliminate the phase ambiguity $\hat{\phi}^{(a,i)}$, then the ratio $\frac{A^2}{\sigma_w^2}$ is improved by 3 dB.

With $d_{orig} = \sqrt{x_0^2 + y_0^2} = z_0 \tan(\hat{\vartheta})$, the aggregate root of the Cramer-Rao bound (RCRB) can be written as follow

$$RCRB = \sqrt{CRB(x_0) + CRB(y_0)}$$

$$= \sqrt{\frac{d_{orig}^2 + z_0^2}{\sigma_w^2} + 2} \sqrt{2 A^2 K}$$

$$= \frac{z_0}{\sqrt{2 A^2 K}} \sqrt{\frac{1}{\tan(\hat{\vartheta})^2} + 2}.$$ 

(23)

For the APC algorithm the computation of the CRB is more convoluted and we do not evaluate it.

B. RMSE of the estimator

Herein, we analyze the aggregate RMSE of the estimator that we define as

$$RMSE = \sqrt{E\{|x_0 - \hat{x}_0|^2\} + E\{|y_0 - \hat{y}_0|^2\}}$$

$$= d_{orig} \sqrt{E\left\{\left(\frac{\tan(\hat{\vartheta})}{\tan(\hat{\vartheta})} - 4 \tan(\hat{\vartheta}) \sin \left(\frac{\hat{\phi} - \phi}{2}\right)\right)^2\right\}}.$$ 

(24)

As (24) shows, the RMSE is a function of the distance from the origin $d_{orig}$ (or equivalently of the height $z_0$ and of the elevation angle $\hat{\vartheta}$), of the error in the estimation of the azimuth $\hat{\phi} - \phi$ and of both the elevation $\hat{\vartheta}$ and its estimation error $\hat{\theta} - \theta$.

In Fig. 2, we report the comparison between the aggregate RMSE for the DPC and the APC algorithms as a function of the SNR for different values of SDR on the left plot and for different values of $\sigma_{\Delta f}$ on the right plot. First, we can see that the RCRB and the simulated curves for the DPC algorithm are in excellent agreement up to about SNR = 50 dB. After this value, the estimator exhibits an error floor for decreasing values of SDR or increasing values of $\sigma_{\Delta f}$, while the RCRB continues to be practically overlapped to the RMSE evaluated in ideal conditions ($SDR = +\infty$ and $\sigma_{\Delta f} = 0$ Hz).

For the curves on the right plot with $\sigma_{\Delta f} = 10$ kHz, the error floor is due to the fact that the carrier frequency offsets among the different antennas are not identical and the algorithm can perfectly compensate them only when they are identical. So, in the presence of different carrier frequency offsets, as more in general assumed in the simulations, there exists a phase error that determines the floor. Finally, we can observe that the RMSEs for the APC algorithm are smaller than the ones of the DPC algorithm. In fact, we can obtain the same RMSE of the DPC algorithm with the APC, but having a lower SNR (about 3 dB less).

In Fig. 3, we report the comparison between the aggregate RMSE as a function of the SNR for different values of SDR on the left plot and for different values of $\sigma_{\Delta f}$ on the right plot, for the APC and the root-MUSIC algorithms. $M = 4$, SNR = 30 dB, SDR = $+\infty$, $\sigma_{\Delta f} = 0$ Hz, $\phi = 45^\circ$.

![Fig. 2. Aggregate RMSE and RCRB as a function of the SNR, SDR (left plot) and $\sigma_{\Delta f}$ (right plot) with the DPC and the APC algorithms. $M = 4$, $x_0 = 30$ m, $y_0 = 30$ m, $z_0 = 10$ m.](image1)

![Fig. 3. Aggregate RMSE as a function of the SNR, SDR (left plot) and $\sigma_{\Delta f}$ (right plot) with the APC and the root-MUSIC algorithms. $M = 4$, SNR = 30 dB, SDR = $+\infty$, $\sigma_{\Delta f} = 0$ Hz, $\phi = 45^\circ$.](image2)
pairs of antenna elements.

We can see (Fig. 3) that in ideal conditions the proposed estimator provides better performance than the root-MUSIC (with $M = 4$). Furthermore, we can observe on the left plot a similar behaviour between the two algorithms in the presence of DC offset, while on the right plot it is shown that the proposed estimator suffers less the presence of the different carrier frequency offsets ($\sigma_{\Delta f} \neq 0$).

In Fig. 4, we consider the aggregate RMSEs as a function of the elevation angle $\vartheta$ on the left plot and of the distance from the axes origin $d_{\text{orig}}$ on the right plot, for two different height values $z_0$, assuming a constant azimuth $\phi = 45^\circ$. These results are obtained with the DPC algorithm, in order to compare them with the RCRBs. With respect to this, the RCRBs are in good agreement with the simulation results. Furthermore, as we can see from (23), with the increase of both the height $z_0$ and the elevation $\vartheta$, the error increases and this behaviour is confirmed by the left plot. We now consider the right plot. In general, for a given value of $z_0$, the positioning error increases for increasing values of $d_{\text{orig}}$, as it can be seen from (23). Now, we can define the critical distance from the origin $d_{\text{orig},c}$ as that for which the curve associated to an array a height $z_0$ and the curve associated to an array at height $z_1$ cross each other. Exploiting (23), we can evaluate $d_{\text{orig},c}$ as

$$d_{\text{orig},c}(z_0, z_1) = \sqrt{\frac{z_0^2 + z_1^2}{z_0}}.$$

For values of $d_{\text{orig}}$ larger than $d_{\text{orig},c}$ (see the right plot in Fig. 4) an increase in the array height is beneficial. This is because as the array height increases the elevation angle $\vartheta$ decreases, for a certain $d_{\text{orig}}$ value, and the error decreases too (see the left plot in Fig. 4). For values of $d_{\text{orig}}$ smaller than $d_{\text{orig},c}$, a decrease in the array height is beneficial. In particular, focusing on the limit case $d_{\text{orig}} = 0$, i.e., $\vartheta = 0^\circ$, it is simple to prove that (24) becomes $\text{RMSE} = z_0 \sqrt{E\{\tan(\vartheta)^2\}}$, therefore it decreases linearly with the height $z_0$ since the expectation term does not depend on the distance between the source and the receiver (that is, $z_0$ in this particular case), once we have set the angles of arrival. As a numerical example, $d_{\text{orig},c}(5m, 10m) = 8.4$ m.

V. CONCLUSION

We have presented an algorithm for positioning in a LOS propagation scenario, based on 2-D AoA estimation, when the receiver height $z_0$ is known. Furthermore, we have considered the effects of several hardware impairments, such as carrier/phase offsets and DC bias. Both the CRB and the simulation analysis show that the proposed estimators are robust for a wide range of distances from the receiver, also in the presence of the impairments, and provide good performance comparable with the those of the well known root-MUSIC. Finally, we have carried out an analysis of the positioning error as a function of the receiver height $z_0$.

REFERENCES


