Frequency Domain Multiuser Detection for Impulse Radio Systems

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Abstract—This paper deals with channel estimation, and detection in ultra wide band (UWB) bi-phase impulse modulated systems. We consider a multiuser scenario assuming a direct sequence spreading code division multiple access (DS-CDMA) scheme. The channel estimation and detection approach operates in the frequency domain and includes the capability of canceling the multiple access interference through the exploitation of its frequency domain correlation. The approach can be extended to time-hopped impulse radio systems.

Keywords—CDMA, Channel estimation, Frequency domain processing, Impulse modulation, Interference Cancellation, Multiuser Detection, UWB systems.

I. INTRODUCTION

This paper deals with synchronization, channel estimation, and detection in impulse radio systems [1]. Several combinations of modulation, and user multiplexing schemes have been proposed for impulse radio communications [2]. The common attractive feature is the carrier-less baseband implementation that involves transmission of short duration pulses. This technology is commonly referred to as ultra wide band (UWB) because the pulses can occupy a very large bandwidth. Most of the work has focused so far on schemes that deploy time hopping spreading codes with pulse position modulation [1]. Instead, in this paper we assume the deployment of bi-phase pulse amplitude modulation (BPAM) in conjunction with direct sequence code division multiplexing of users (DS-CDMA) [2]-[4]. Binary codewords are assigned to users and modulate short duration pulses (monocycles). A user’s codeword spans a transmission frame. Frames are separated by a guard time to cope with the time dispersion that is introduced by the channel frequency selectivity.

When the guard time is longer than the channel time dispersion, and only a single user accesses the medium, the optimal receiver comprises a matched filter followed by a symbol by symbol threshold detector [5]. The receiver filter has to be matched to the equivalent impulse response that comprises the user’s waveform, and the channel impulse response. Since UWB signals can occupy a large bandwidth, the channel is highly frequency selective and the received signal exhibits a large number of multipath components. Potentially, high frequency diversity gains can be achieved [5]. However, the optimal matched filter receiver has to accurately estimate the channel, and such an estimation can be particularly complex if performed in the time domain. It has been shown in [6] that channel estimation can be partitioned into a two step process if we model the channel as a tapped delay line. That is, we can first determine the channel ray delays, and then we can obtain an estimate of the ray amplitudes. Unfortunately, the ray search has a complexity that grows exponentially with their number. Further, false ray detection may occur in the absence of a priori knowledge about the true number of rays. Such a search can be partially simplified under the assumption of the channel to be separable [6]-[7]. However, this assumption can translate into deep performance losses in the non-rare event of clusters of non-resolvable rays.

It has also to be emphasized that when the common media is shared by multiple users, multiple access interference (MAI) may arise at the receiver side. In a DS-CDMA system, this is due to the deployment of non orthogonal codes, or to users that are time asynchronous, or to the presence of channel time dispersion. Assuming a single user detection approach the MAI translates into performance losses, such that some form of multiuser detection is advisable [3].

Motivated by the above considerations, we propose in this paper a frequency domain approach to channel estimation, and detection. The approach includes the capability of rejecting the MAI interference. It has been derived from the observation that the optimal matched filter receiver can be equivalently implemented in the frequency domain. The approach comprises the following stages (Fig. 1). First we acquire frame synchronization with the desired user. Second, we run a discrete Fourier transform (DFT) on the received frames. Third, we perform frequency domain channel estimation for the desired user via a recursive least squares (RLS) algorithm. Finally, detection is accomplished in the frequency domain.

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1 This work was partly supported by MIUR under project FIRB-PRIMO "Reconfigurable platforms for wideband wireless communications".

Proceedings of IEEE Vehicular Technology Conference Spring 2005
Stockholm – Sweden
May 30 - June 1, 2005
using the estimated channel frequency response. Frame synchronization is required and it is described in [8].

In the presence of multiple access interference its rejection is accomplished by observing that the MAI manifests itself with a frequency domain correlation. Such a correlation can be estimated with a simple least square approach, and it can be exploited by the frequency domain detector.

II. IMPULSE MODULATED SYSTEM MODEL

In our system model (Fig. 1) we assume bi-phase pulse amplitude (BPAM) modulation such that the signal transmitted by user \( u \) can be written as

\[
s^u(t) = \sum_{k=-\infty}^{\infty} b_k^u g^u(t - kT_f) \]

where \( b_k^u \) denotes the information bit transmitted in the \( k \)-th frame, \( g^u(t) \) is the waveform used to convey information for user \( u \), and \( T_f \) is the bit period (frame duration). We further deploy direct sequence spreading to accommodate for multiplexing of users. The user's waveform (signature code) comprises the weighted repetition of \( L \geq 1 \) narrow pulses (monocycles), i.e.,

\[
g^u(t) = \sum_{m=-\infty}^{L-1} c_m^u g_{su}(t - mT) \]

where \( c_m^u \) are the codeword elements (chips) of user \( u \), and \( T \) is the chip period. We can choose the codewords to be either orthogonal or random (pseudo-noise). We incorporate the differential effect of the transmit-receive antennas into \( g_{su}(t) \), and we assume it to be the second derivative of the Gaussian pulse, \( g_{su}(t) \sim \exp(-\pi/(2((t - D/2)/T_o)^2)) \). In typical system design we can choose \( T_o \geq 2T_s \) is the monocyte pulse duration. We further insert a guard time \( T_g \) between frames to cope with the channel time dispersion, and eliminate the inter-symbol interference (ISI). The frame duration fulfills the relation \( T_f > L T + T_g \) with \( T_g \) being the channel time dispersion.

As shown in Fig. 1, at the receiver side we first deploy a band-pass front-end filter with impulse response \( g_{RF}(t) \) to suppress out of band noise, and interference. Then, the received signal in the presence of \( N_f \) other users (interferers), reads

\[
y(t) = \sum_{k=0}^{N_f} b_k g_{RF}(t - kT_f) + \eta(t) \]

where \( \eta(t) \) is the additive white Gaussian noise with zero mean, and doubly sided power spectral density \( N_o/2 \). The equivalent impulse response comprises the convolution of the \( u \)-th user's transmission waveform (signature code) with its channel impulse response, and the front-end filter. Distinct users experience independent channels that we assume to introduce identical maximum time dispersion. The channel impulse response is assumed to be time-invariant over several transmitted frames. Then, it can change in a random fashion. With the popular discrete multi-path model [6]-[7], the channel impulse response of user \( u \) can be written as

\[
h^u(t) = \sum_{p=1}^{P_u} a_p^u \delta(t - \tau_p^u). \]

As an example, the tap delays are drawn from a uniform or a Poisson process, while the tap gains are assumed to be real, independent, and equal to \( a_p^u = \chi_p^u \beta_p^u \) with \( \beta_p^u \) Rayleigh/Log-normal distributed, while \( \chi_p^u \) takes on the values \( \pm 1 \) with equal probability. The rays can appear in clusters of duration less than \( D \), i.e., they are not necessarily resolvable.

III. FREQUENCY DOMAIN PROCESSING

The conventional correlation receiver (matched filter receiver) operates in a symbol by symbol fashion by computing the correlation between the received signal frame \( y_k(t) = y(t + kT_f) \), \( 0 \leq t < T_f \), and the real equivalent impulse response \( g_{RF}(t) \) to obtain \( x(kT_f) = \int_0^{T_f} y_k(t) g_{RF}(t)dt \). Then, a threshold decision is made to detect the \( k \)-th transmitted bit, i.e.,

\[
\hat{b}_k = \text{sign} \{ x(kT_f) \} \]

To implement the correlation receiver in the time-domain we need to estimate the channel impulse response. Time-domain channel estimation is complicated by the high number of multipath components exhibited by UWB channels, and by the presence of non resolvable channel rays, i.e., rays with relative time delay smaller than the monocyte duration \( D \). Thus, \( g_{RF}(t) \) can be an involved function of the channel, and the transmitted waveform.

In this paper we propose to perform channel estimation, and detection in the frequency domain. We assume discrete-time processing, such that the received signal is sampled at the output of the front-end analog filter at sufficiently high rate. We acquire frame synchronization with the desired user. Then, we run an \( M \)-point discrete Fourier transform (DFT) over the \( M \) samples of the \( k \)-th frame \( y_k(nT_f) \), \( T_f = T_o / M \), to obtain

\[
Y_k(f_s) = b_k G_{RF}(f_s) + I_k(f_s) + N_k(f_s) \]

where \( Y_k(f_s), G_{RF}(f_s), I_k(f_s), N_k(f_s) \), for \( f_s = n/MT_f \), are respectively the DFT outputs of the received frame samples, the desired user equivalent impulse response, the interference, and the noise samples. No ISI is present for the desired user assuming perfect frame timing, and a sufficiently long guard time. The MAI term with asynchronous-synchronous users is a function of the users' time delay, transmitted waveform, and channel. In the asynchronous case two information bits per user may cause interference, while in the synchronous case only one bit generates interference.

To derive the proposed receiver, we model the noise-plus-interference \( z(kMT_f + nT_f) = z_k(nT_f) = i_k(nT_f) + n_k(nT_f) \) with a discrete time colored Gaussian process with correlation function \( r(nT_f, MT_f) = E[z(nT_f)z^*(MT_f)] \). Then, assuming the transmitted bits of all users to be i.i.d. and equally likely, the DFT outputs \( Z_k(f_s) = I_k(f_s) + N_k(f_s) \) are complex Gaussian with zero mean.

The impairment multivariate process \( Z_k \) defined as \( Z_k = [Z_k(f_s), ..., Z_k(f_{SU})]^T \) has time-frequency correlation
matrix equal to \( R(k,m) = E[Z,Z^\dagger] = FK(k,m)F^\dagger \) \(^{(6)}\),

where \( K(k,m) \) is the \( M \times M \) matrix with entries \( r(k+m+n,mM+l) \) for \( n,l = 0,...,M-1 \), and \( F \) is the M-point DFT orthonormal matrix. With uncorrelated thermal noise samples, \( R(k,m) = 0 \) for \( |m-k| > 1 \) in the asynchronous MAI case, while \( R(k,m) = 0 \) for \( |m-k| > 0 \) in the synchronous case. Now, let us collect the elements of \( Y_i(f_i) \), and of \( G_{EQ}(f_i) \) in the vectors \( Y_i \) and \( G_{EQ} \). Then, we show in the Appendix that under the above colored Gaussian impairment model, the maximum-likelihood receiver can be equivalently implemented in the frequency domain by searching the sequence of transmitted bits \( \hat{b}_i \) (belonging to the desired user) that maximizes the log-likelihood function:

\[
\Lambda(\hat{b}_i) = -\sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} [Y_i - \hat{b}_i G_{EQ}(k,m)] R^{-1}(k,m) [Y_i - \hat{b}_i G_{EQ}(k,m)]^\dagger. \quad \text{(7)}
\]

In order to simplify the algorithm complexity we neglect the temporal correlation of the interference (MAI+noise) vector \( Z_i \), i.e., we assume \( R(k,m) = 0 \) for \( k \neq m \). Indeed, the MAI correlation across frames is zero only for the synchronous case. Then, by dropping the terms that do not depend on the information bit of the desired user, the log-likelihood function simplifies to

\[
\Lambda(\hat{b}_i) = -\hat{b}_i \text{Re}\{G_{EQ}^\dagger(k,k)Y_i\}. \quad \text{(8)}
\]

Therefore, according to (8) the frequency domain receiver operates on a frame by frame basis, and it exploits the frequency correlation of the MAI. The computation in (8) can be interpreted as the result of matching the frequency response of the \( k \)-th frame with \( G_{EQ}^\dagger(k,k) \) to obtain

\[
z_{kk}(kT_f) = G_{EQ}^\dagger(k,k)Y_i. \quad \text{(9)}
\]

Then, we make a decision on the transmitted bit looking at the sign of (9). Note that (9) is real, given that the quantities involved have Hermitian symmetry.

In the absence of MAI, and with white noise, the correlation matrix is diagonal with diagonal elements equal to the noise variance. We assume the correlation matrix to be full rank, otherwise pseudo-inverse techniques can be used.

To obtain (8) we need to estimate \( G_{EQ}(f_i) \). The attractive feature with this approach is that the matched filter frequency response at a given frequency depends only on the channel response at that frequency. This greatly simplifies the channel estimation task. By exploiting the Hermitian symmetry of \( G_{EQ}(f_i) \), the estimation can be carried out only over \( M/2 \) frequency bins. A further simplification is obtained by observing that the desired user’s waveform can be written as

\[
G(f) = Gd(f) \sum_{n=0}^{N-1} c_n e^{-j2\pi fn/Tc}.
\]

If we deploy a monochrome that has a frequency concentrated response, as the Gaussian pulse, we can assume that \( G_{d}(f_r) = 0 \) for, say, \( f_r > 2/D \). Therefore, relevant signal energy is present only in a small number of frequency bins, and consequently channel estimation can be performed only over this fraction of bins. Another interesting characteristic of the frequency domain channel estimation approach is that no restrictive assumption about the channel impulse response has been made.

### A. Frequency Domain Parameter Estimation

To estimate the frequency response of the desired user channel, and the interference correlation matrix we assume the deployment of a training sequence of \( N \) known bits. To keep it simple, we run estimation in a two steps procedure. First, we estimate the desired user’s channel. Then, we estimate the interference correlation matrix. We implicitly assume the channel, and the MAI to be stationary over the transmission of several frames, i.e., \( R = R(k,k) \). In particular, the \( M \)-bins channel frequency response can be obtained via a recursive least squares (RLS) algorithm that operates independently over the sub-channels [8]-[9]. Once we have computed the desired user’s frequency domain channel estimate \( \hat{G}_{EQ} \), we compute an estimate of the interference correlation matrix \( \hat{R} \). Let us define the error vector in correspondence with the \( i \)-th frame as

\[
\hat{E}_i = b_i Y_i - \hat{G}_{EQ} \quad \text{where} \quad |b_i|, \quad i = 0,...,N-1,
\]

is the sequence of known training bits of the desired user. Then, we estimate the correlation matrix as \( \hat{R} = 1/N \sum_{i=0}^{N-1} \hat{E}_i \hat{E}_i^\dagger \). Further, to introduce a tradeoff between the effects of noise, and the effects of the MAI we add diagonal loading as follows:

\[
\hat{R} = (1-\rho)\hat{R} + \rho\sigma_i^2 I,
\]

with \( \rho \leq 1 \) and \( I \) being the identity matrix. For practical purposes the noise variance can be set to an appropriate value according to the range of operating signal-to-noise ratios.

### B. Frame Synchronization

Frame synchronization with the desired user is acquired in the time domain and uses the training bit sequence. We use a two steps method where we first determine a coarse estimate of where the received signal energy is. Then, we refine timing. Details can be found in [8].

### IV. PERFORMANCE RESULTS

The performance of the proposed algorithm is assessed via simulations. The sampling period \( T_s \) is taken as the time unit, while the frame duration is \( T_f = MT_c \), with \( M=256 \). We use three statistical channel models for the response (4):

**Channel A.** It has \( N_p=10 \) rays that have uniform delay distribution within \([0, T_f - D] \) and Rayleigh amplitude with equally likely sign flip. \( T_s \) is the frame duration while \( D \) is the monocycle duration. The rays have average power

\[
\Omega_p = E[\sigma_p^2] \approx e^{-p/D}, \quad p = 0,...,N_p-1.
\]

**Channel B.** The ray delays belong to the interval \([0, T_f - D]\) and are drawn from a Poisson process with mean arrival rate \( \Lambda / D \). The ray amplitudes are log-normal distributed with equally likely sign flip. The first ray is fixed at time instant 0.
The rays have power \( \Omega_p = e^{-\tau_p / (4D)} \) for a given decay factor \( \Gamma \), and mean \( E[\alpha_p] = 0.5 \sqrt{\Omega_p} \). Note that \( \Lambda \) represents the mean number of rays per monocycle in the underlying arrival Poisson process.

**Channel C.** It is like Channel A with \( N_p = 5 \) rays in \([0, T_f - 8D)\), with power \( \Omega_p = e^{-\tau / (4.5D)} \).

### A. Single User Performance

In Fig. 2 we evaluate the performance of the channel estimator in terms of BER as a function of the length of training sequence assuming perfect frame timing. A single user with no spreading \((L=1)\) transmits over the Channel A model. We assume \( D = 63T_c \). The convergence of the practical estimator to the performance that is achieved with ideal matched filtering, is fast. We report both the performance that is obtained when we combine all frequency bins (curves labeled with 100% Freq. Bins in Fig. 2), and the performance that is obtained when we combine only the bins for which \( |G(f)_{ij}| > 0.1 \times \max_x |G(f)| \) (curves labeled with 10% Freq. Bins). We have found that with ideal matched filtering there is no appreciable performance difference. However, with practical estimation we actually improve performance. This is because the estimation over frequency bins that have small signal energy is poor and can negatively affect the BER performance. For the 10% Freq. Bins curves, channel estimation is performed only over 17 bins out of 256.

We emphasize that the proposed frequency domain receiver does not rely on a particular statistical channel model. Clearly, its benefit compared to the rake receiver in the single user case is a function of how resolvable the channel is. To show this point, we report in Fig. 3 the BER performance for the Channel Model B in a single user system with no spreading. In particular in Fig. 3A we fix the SNR to 8 dB and we plot the BER as a function of the average number of rays per monocycle \( \Lambda \), fixing \( \Gamma = 4.75 \). In Fig. 3B we plot the BER as a function the normalized decay factor \( \Gamma \), fixing \( \Lambda = 4.75 \). Furthermore, the simulation assumes \( D = 25T_c \). The channel can span about 9 monocycles. Training has length 100 bits. As a comparison, we plot the performance of the rake receiver that combines up to 8 resolvable fingers, i.e., spaced by at least \( D \). To allow for practical implementation the ray search is independently done as described in [7]; we look for the resolvable maxima at the output of the front-end filter that is matched to the monocycle. This is because the optimal joint ray search algorithm in [6] is prohibitively high. This rake receiver is very simple although suboptimal. Now, Fig. 3A shows that as the average number of rays per monocycle increases the performance of the FD channel estimator increases while the rake receiver actually worsens. This is because the rake receiver is not capable of capturing the whole channel energy and fully exploit diversity. For the same reason also in Fig. 3B the rake receiver performs worse than the FD algorithm as the decay factor (delay spread) increases.

### B. Multiuser Case

The performance of the proposed algorithm in a multiuser scenario is shown in Fig. 4. We deploy random (pseudo noise) short codes of length \( L = 8 \) for all users. This allows us to keep the simulation runtime within moderate values. Longer codes shall yield improved performance.

The chip period is set to \( T = D = 25T_c \). The training sequence has length \( N = 150 \) bits. The users experience independent channels according to the Model C that we have described before. Both the synchronous case (dashed curves) and the asynchronous users case (solid curves) are considered. For the asynchronous case, the users’ time delays are independent and uniformly distributed within a frame interval. The signal-to-noise ratio is set to \( E_s / N_0 = 12 \) dB with equal power users. Fig. 4 shows that a sensible performance degradation arises in the presence of multiple users if no MAI cancellation is performed. Note that we simulate also an overloaded system scenario, i.e., we allocate more than \( L = 8 \) users. Curves labeled with ideal have been obtained assuming ideal knowledge of the channel and frame timing of the desired user, while the curves labeled with practical have been obtained by estimating both the frame timing [8], and the channel (with the frequency domain RLS algorithm). Performance can be significantly improved by deploying the proposed frequency domain MAI canceling algorithm. In both the ideal, and the practical case the interference correlation matrix has been estimated over the training sequence of length 150 bits using diagonal loading with a loading factor 0.5. This is to improve the performance with a small number of users, i.e., when performance is dominated by the noise rather than by the MAI. Furthermore, we combine only the frequency bins that have amplitude above 10% the maximum of \( |G(f)| \). In our simulations this resulted into at most 82 overall frequency bins. Fig. 4 shows that the performance of the correlation receiver that does not cancel the MAI rapidly decreases as the number of users increases. On the other hand the canceling algorithm performs well, and exhibits a much flatter curve slope. We point out that the training parameters have been kept fixed for all scenarios. Indeed, further improvements are possible by optimizing the parameters.

### V. CONCLUSIONS

We have proposed a frequency domain processing approach to detection, channel estimation, and MAI cancellation for impulse radio CDMA systems. The approach can be extended to time-hopping based systems.

**APPENDIX : DERIVATION OF THE DETECTION METRIC**

Let us start from the (real) discrete time received signal model in (3) where the impairment (interference plus noise) \( z(nT_c) = i(nT_c) + n(nT_c) \) is modeled with a zero mean colored Gaussian process (non necessarily stationary) with correlation function \( r(nT_c, lT_c) = E[z(nT_c)z(lT_c)] \). With this model, under the knowledge of the channel, the maximum likelihood receiver searches for the sequence of transmitted bits \( \{b_i\} \) (belonging to the desired user) that maximizes the logarithm of the probability density function of the received signal \( y = [\ldots, y(0), y(T_c), \ldots] \) conditional on a given hypothetical transmitted bit sequence, i.e., \( \log p(y | \{b_i\}) \). It follows that we have to search for the bit sequence of the desired user that maximizes the following log-likelihood function [10]:

\[ p(y | \{b_i\}) = \prod_{n=0}^{N} p(y(nT_c) | b_i) \]
\[ \Lambda(\{\hat{b}_k\}) = -\sum_{l=-M/2}^{M/2-1} \sum_{m=0}^{M-1} \left( y(lT_e \phi) - \sum_k \hat{b}_k g_{EQ}(lT_e - kT_r) \right) \times \mathbf{K}_{l,m}^{-1} \left( y(mT_e \phi) - \sum_k \hat{b}_k g_{EQ}(mT_e - kT_r) \right) \]

where \( \mathbf{K}_{l,m} \) is the element of indices \((l,m)\) of the inverse of the matrix \( \mathbf{K} = \mathbb{E}[\mathbf{z}\mathbf{z}^H] \) with \( \mathbf{z} \) being the time-domain impairment vector. If we define the vector \( \mathbf{e} = [\ldots, e(0), e(T_e), \ldots,] \), with \( e(lT_e \phi) = y(lT_e \phi) - \sum_k \hat{b}_k g_{EQ}(lT_e - kT_r) \), the above function can be written as the following scalar product \( \Lambda(\{\hat{b}_k\}) = -\mathbf{e}^H \mathbf{K}^{-1} \mathbf{e} = -\mathbf{e}^H \mathbf{F}^{-1} \mathbf{e} \). Since the scalar product is irrelevant to an orthonormal transform (Parseval theorem), we have that \( \Lambda(\{\hat{b}_k\}) = -\mathbf{e}^H \mathbf{F}^{-1} \mathbf{e} \) with \( \mathbf{F} \) being the block diagonal orthonormal matrix that has blocks all identical to the \( M \)-point DFT matrix \( \mathbf{F} \). Since with our assumptions \( g_{EQ}(\nu T_e \phi) \) has support in \([0, T_e \phi]\), the vector \( \mathbf{e} = \mathbf{F} \mathbf{e} \) can be partitioned into non overlapping blocks equal to \( \mathbf{E}_k = \mathbf{Y}_k - \hat{\mathbf{b}}_k \mathbf{G}_{EQ}, \) i.e., the \( M \)-point DFT of the \( k \)-th received frame minus the DFT of the hypothesized signal. It follows that

\[ \Lambda(\{\hat{b}_k\}) = -\mathbf{e}^H \mathbf{F} \mathbf{K}^{-1} \mathbf{F} \mathbf{e} = -\mathbf{e}^H \mathbf{R}^{-1} \mathbf{e} \]

where we have used the identity \( \mathbf{F} \mathbf{K}^{-1} \mathbf{F}^{-1} = \mathbb{E}[\mathbf{z}\mathbf{z}^H] \mathbf{F}^{-1} = \mathbf{R} \), and \( \mathbf{R}^{-1}(k,m) \) denotes the \( M \times M \) block of indices \((k,m)\) of \( \mathbf{R}^{-1} \). The above result proves the metric in (7).

### REFERENCES


