Short LDPC Codes for NB-PLC Channel with a Differential Evolution Construction Method

Nikoleta Andreadou and Andrea M. Tonello
Department of Electrical, Management and Mechanical Engineering
University of Udine – Via delle Scienze 208
33100 Udine – Italy
E-mail: {nikoleta.andreadou, tonello}@uniud.it

Abstract – In this paper, we study the performance of Low Density Parity Check (LDPC) codes and we introduce an optimized version of these codes when applied on the short data blocks of a narrowband (NB) power line communications (PLC) system. We compare their performance with the one obtained when the G3 standard coding scheme is applied on the system, namely the scheme that concatenated convolutional with Reed-Solomon codes. Multipath propagation together with two noise scenarios are considered: background AWGN noise with and without the presence of impulsive noise. The results indicate that under the considered channel and noise conditions the introduced optimized LDPC codes can operate better than the concatenated scheme used in the G3 standard. The optimized irregular LDPC codes are obtained from the well-established method of differential evolution for finding a degree distribution pair that improves the performance. The evaluation of each candidate code, is proposed to be carried out with a method that takes into account the actual channel and noise conditions. The equivalent optimized degree distribution pairs are presented.

Index Terms—NB-PLC Channel, LDPC codes, Short-Block length, Differential evolution.

I. INTRODUCTION

Power Line Communication is a technology that has greatly gained the scientific interest recently. It refers to the use of the existing power line grid for telecommunication signal transmission. Narrowband (NB) PLC, which is also the research topic in this work, is the evolving PLC technology that occupies a narrow frequency band. NB-PLC has been developed with the scope of offering indoor (home automation) and outdoor (smart grid) command and control services. Standards for NB-PLC have been made available just in 2011, like the G.HNEM standard [1]. The PRIME [2] and the G3 specifications [3], have been the basis for NB-PLC standardization. In [4], [5] a physical layer oriented analysis of the existing standards is provided, whereas in [6] MAC layer mechanisms are studied. Channel and noise issues are studied in [7], [8]. Coding techniques are presented in [9]-[11].

Since the PLC channel is a hostile environment for the telecommunication signal, encoding becomes an important research topic, since it provides data protection. In [11], it is shown that, when the AWGN channel is used as a transmission medium, a class of (LDPC) codes, namely the Quasi Cyclic (QC)-LDPC codes with 512 and 1024 code length can outperform the concatenated scheme with convolutional and Reed Solomon codes used in the G3 standard.

Therefore, LDPC codes could be considered as a candidate scheme for the narrowband PLC case when short data blocks are transmitted. These codes have been firstly introduced by Gallager [12], and they are described by their parity check matrix. In [13], [14] a method, called density evolution, has been introduced for the AWGN channel in order to find the optimized version of irregular LDPC codes. It is a numerical method of evaluating the probability of error of an ensemble of codes through the densities of the messages exchanged during the decoding procedure. Later on, it has been combined with the method of differential evolution [15], which is a heuristic method for finding a good degree distribution pair characterizing the LDPC matrix. This procedure has also been applied on the Rayleigh fading [16] and the erasure channel [17] for the LDPC code optimization. The concept of density evolution is that it assesses each code performance via the resulting probability of error, whereas it requires the statistics of the channel/noise in order to initialize the procedure. In the case of the NB-PLC channel, this is not directly applicable since the statistics of the channel and noise cannot be defined in closed form or give rise to a mathematically convoluted problem. Therefore, in this paper, we implement a more practical yet taking into account the characteristics of the noise and PLC channel, thus carrying out the code optimization under realistic conditions.

First of all, we study the case of LDPC codes with different code lengths in the NB-PLC channel. It is assumed that the channel has a multipath nature and two different noise scenarios are implemented: AWGN background noise with and without the presence of impulsive noise. The OFDM transmission technique is also considered. We compare the performance of LDPC and the concatenated scheme that uses convolutional with Reed Solomon codes. Further on, we apply the well-established procedure of differential evolution for finding a good code degree distribution pair. An approach is incorporated for the evaluation of each tested code that takes into account the actual channel and noise conditions. For this purpose, many channel realizations are taken into account and the average code performance is computed. The degree distribution pair that results in the best average performance is selected for the LDPC matrix construction of each considered code length. The contribution of the paper is summarized as follows:

The work of this paper has been partially supported by the POR/FESR, FVG, Italy, 2007-2013 programme, project LAK - Living for All Kitchen.
We consider transmission on a NB-PLC channel affected by multipath fading and impulsive noise. The transmission is accomplished by the OFDM technique.

We report several performance results for different noise conditions, two different code techniques with various code lengths.

We use the differential evolution to find optimized degree distribution pairs of LDPC codes along with a practical approach to evaluate each candidate code performance.

The rest of the paper is organized as follows. Section II describes the coding and transmission techniques used, while in Section III the channel and noise specifications are presented. In Section IV, the coding schemes are implemented on the PLC channel and their performance is illustrated. In Section V, the optimization technique is presented and the results are shown. Conclusions are drawn in Section VI.

II. CODING AND TRANSMISSION TECHNIQUES

A. Coding Scenario A: Concatenation of Convolutional and Reed Solomon Codes

The concatenation of convolutional and Reed Solomon codes, which is also used in the G3 standard [3], is chosen as the first coding scenario for the system realization. In general, the physical layer specifications indicated in [3] are followed. Information data is firstly encoded with a Reed Solomon encoder using 8 bits to form a symbol. A convolutional encoder follows next with a code rate of ½ and encoder using 8 bits to form a symbol. A convolutional encoder to all zero states. The decoding is performed through the so-called Sum-Product decoding algorithm. It is characterized by extrinsic messages sent between variable and check nodes in the bipartite graph [14]. Seven different code and packet lengths are here employed and tested in the system. Table II shows the equivalent data block specifications for code scenario B, whereas in Fig. 2 the system block diagram is presented. The LDPC codes are used with a code rate of ½. It is noteworthy that no interleaver is implemented for this case, since the nature of LDPC decoding algorithm is such that can cope up for its use. In OFDM, the useful data carriers are again 36 and the number of OFDM symbols used to carry all the codeword bits differs for each data block length. In addition, the total resulting code rate is somehow greater than the overall code rate regarding the baseline code concatenated scenario. The difference becomes larger for smaller data blocks, and is due to the code rate of the Reed Solomon codes introduced in the concatenated version. For the decoding, the number of iteration rounds is set to 35.

B. Coding Scenario B: QC-LDPC Codes

LDPC codes belong to a subgroup of Linear Block Codes. These codes are characterized by their parity check matrix, \( H \), which is sparse. A code rate of \( k/n \), results in a parity check matrix with size \((n-k) \times n\). The LDPC parity check matrix can be described via a bipartite graph, which comprises the check and bit nodes. Each check node stands for each row in the parity check matrix, while each bit node represents a codeword bit. The nodes are connected to each other in case there is an ace in the corresponding position of the parity check matrix [14].

As a first approach, the class of non-optimized LDPC codes used in this paper is the QC-LDPC codes, characterized by a parity check matrix consisting of square blocks. These blocks could either be circulant permutation matrices based on the identity matrix or zero matrices. All non-zero permutation matrices \( P \) of size \( q \times q \) are derived from the identity matrix \( I \), after shifting its columns to the right by \( i \) times \((0 \leq i \leq q)\). We focus on a matrix in upper triangular form [18]:

\[
H(q,j,k) = \begin{bmatrix}
1 & 1 & \ldots & 1 & \ldots & 1 \\
0 & I & \ldots & p_{(j-2)} & \ldots & p_{(k-2)} \\
0 & 0 & I & \ldots & p_{(j-3)} & \ldots & p_{(k-3)} \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & 0 & I & \ldots & p_{(j-k)}
\end{bmatrix}
\]  

(1)

The decoding is performed through the so-called Sum-Product decoding algorithm. It is characterized by extrinsic messages sent between variable and check nodes in the bipartite graph [14]. Seven different code and packet lengths are here employed and tested in the system. Table II shows the equivalent data block specifications for code scenario B, whereas in Fig. 2 the system block diagram is presented. The LDPC codes are used with a code rate of ½. It is noteworthy that no interleaver is implemented for this case, since the nature of LDPC decoding algorithm is such that can cope up for its use. In OFDM, the useful data carriers are again 36 and the number of OFDM symbols used to carry all the codeword bits differs for each data block length. In addition, the total resulting code rate is somehow greater than the overall code rate regarding the baseline code concatenated scenario. The difference becomes larger for smaller data blocks, and is due to the code rate of the Reed Solomon codes introduced in the concatenated version. For the decoding, the number of iteration rounds is set to 35.

C. OFDM Technique

As mentioned above, in each OFDM symbol, only 36 carriers entail useful information. To be more precise, the frequencies used are in the CENELEC A band, with the first carrier at 35.938 kHz and the last carrier at 90.625 kHz [3]. A codeword is spanned over a number of OFDM symbols, each one using a 256-point IFFT. A cyclic prefix of 30 samples length is also inserted in the symbol to mitigate the effect of intersymbol interference. The OFDM samples to be transmitted are formed as follows:

\[
x_s(l) = \sum_{k=0}^{N-1} b_s(k) \cdot e^{j2\pi k l/N}
\]  

(2)
TABLE II. SPECIFICATIONS ON THE ARRANGEMENT OF DATA IN BLOCKS WITH CODE SCENARIO B

<table>
<thead>
<tr>
<th>Number of OFDM symbols</th>
<th>LDPC encoder: Input/Output bits</th>
<th>Codeword length</th>
<th>Data bits in codeword</th>
</tr>
</thead>
<tbody>
<tr>
<td>112</td>
<td>2010/4020</td>
<td>4032</td>
<td>2010</td>
</tr>
<tr>
<td>56</td>
<td>1005/2010</td>
<td>2016</td>
<td>1005</td>
</tr>
<tr>
<td>52</td>
<td>936/1872</td>
<td>1872</td>
<td>936</td>
</tr>
<tr>
<td>40</td>
<td>715/1430</td>
<td>1440</td>
<td>715</td>
</tr>
<tr>
<td>32</td>
<td>570/1140</td>
<td>1152</td>
<td>570</td>
</tr>
<tr>
<td>20</td>
<td>360/720</td>
<td>720</td>
<td>360</td>
</tr>
<tr>
<td>12</td>
<td>216/432</td>
<td>432</td>
<td>216</td>
</tr>
</tbody>
</table>

The parameter \( N \) is the number of carriers, \( b_k \) is the \( k \)-th bit in the BPSK modulated word included in the \( l \)-th OFDM block and \( x_n \) is the \( n \)-th sample of the transmitted sequence. This sequence is then fed to the PLC channel for transmission.

III. CHANNEL AND NOISE REPRESENTATION

A. Channel Model

The signal undergoes multipath fading introduced by the channel. The statistical channel model used is similar to that described in [19]. However, the bandwidth used in this case is 0-500 kHz, which represents the narrowband PLC channel. The channel frequency response is considered to be:

\[
g_{CH}(f) = \sum_{n=-\infty}^{\infty} \alpha_n \exp(-j2\pi f / v_p) \cdot \exp(-j2\pi f / v_p), \quad 0 \leq f \leq B
\]

(3)

The number \( N_P \) of multipath components is obtained from a Poisson distribution with path rate per unit length equal to \( \Lambda = 0.2 \) and maximum path length \( d_{\text{max}} = 800 \) m. The parameters are defined as: \( K = 1, \alpha_0 = 0.3 \cdot 10^2, \alpha_1 = 4 \cdot 10^{-10}, v_p = 2 \cdot 10^8 \), while the path gains \( \alpha_n \) are uniformly distributed in \([-1, 1]\). The dispersion caused by the channel is such that the cyclic prefix inserted by the OFDM modulator can prevent the effect of intersymbol interference.

B. Noise Scenario A

As a first scenario, the channel noise \( n_b(t) \) is considered to be AWGN with zero mean and variance \( N_0 \).

C. Noise Scenario B

In a second scenario, the noise present in the PLC channel is considered to be comprised of two components: the background and the impulsive noise. The background noise is still considered AWGN. On the other hand, the impulsive noise component is thought to occur in bursts, meaning that some OFDM data blocks are affected by its presence while others are not. The sequence of impulse bursts is defined by three parameters: \( d_{\text{mm}}, A_m \) and \( M_m \), the impulse duration, the burst power amplitude and the interarrival time within impulses respectively. The duration of impulse bursts is uniformly distributed in \([10 \mu\text{sec} - 1 \text{msec}]\). The interarrival time is exponentially distributed with parameter \( \lambda = 0.05 \). The smaller the interarrival time becomes, the more frequent the impulses, thus resulting in severer impulsive noise and inferior system’s performance. The amplitude is considered to be Gaussian with zero mean. In this work we assume that the impulse burst amplitude is such that results in a power of impulsive noise \( N_I = 10 \cdot N_0 \), with \( N_0 \) the power of the background noise.

It follows that the received signal can be written as:

\[
y(t) = x(t) + g(t) + n_b(t) + n_I(t)
\]

(4)

where \( n_I(t) \) is the impulsive noise. This noise scenario is more realistic with respect to the actual conditions on the PLC channel, since impulsive noise does occur on the medium influencing data blocks.

IV. CODES PERFORMANCE ON NB-PLC CHANNEL

In this section, we present the performance of the non-optimized LDPC codes (Section II.B), as well as that of the concatenated convolutional plus Reed Solomon codes on the NB-PLC channel (Section II.A). Fig. 3 shows the performance of the baseline coding scenario under the two described noise conditions, background noise without and with the presence of impulsive noise. Fig. 4 shows the performance of the non-optimized LDPC codes when impulsive noise is absent and present in the system. It is noticeable from Fig. 3 and Fig. 4 that all the packet lengths (i.e. size: 112 stands for the packet length that entails 112 OFDM symbols) follow the same trend. As it can be seen, the concatenated convolutional plus Reed Solomon codes outperform the non-optimized LDPC codes, except for the case of 12 OFDM symbols forming a single codeword. This is explained since for this packet length, the redundancy introduced by the baseline coding scenario is greater than that of LDPC codes; therefore, the resulting \( E_b/N_0 \) curve performance appears to be inferior. It is also noticeable that when impulsive noise occurs, the performance of all code scenarios deteriorates, which is rather expected, since impulses have a negative effect on information data.

V. LDPC CODE OPTIMIZATION THROUGH DIFFERENTIAL EVOLUTION

A. Differential Evolution

It has been proved that the performance of an ensemble of LDPC codes can be enhanced, if the degree distribution pair used to produce their parity check matrix is optimized [14]. This motivates our further research. As it has been mentioned in the Introduction, the two popular methods of optimizing the LDPC codes for a variety of channels [14], [16], [17] are density and differential evolution performed in an iterative manner. The concept is that the latter method provides with a candidate degree distribution pair corresponding to an LDPC code. The density evolution method requires an analytic formulation of the BER (Bit Error Rate) from an analytic knowledge of the channel statistics, while it uses the evolution of message densities processed during the decoding algorithm. However, the statistics of the NB-PLC channel cannot be defined in closed form or give rise to a mathematically convoluted problem. Therefore, in this work, the differential evolution is utilized for the derivation of a candidate degree distribution pair, whereas the performance it
provides is evaluated through a more practical approach. Each code is tested under representative conditions of the NB-PLC channel. For this purpose, many channel realizations are taken into account and the average performance is the one that characterizes each code degree distribution pair.

The number of channel realizations \( n_{ch} \) is a function of the code length used each time. The objective is to find the pair of degree distributions such that the probability of error diminishes after a number of iterations for the decoding algorithm. For a degree distribution pair \( \lambda(x) \) and \( \rho(x) \), we can write

\[
\lambda(x) = \sum_{j=2}^{\infty} \lambda_j \cdot x^{j-1} \\
\rho(x) = \sum_{j=2}^{\infty} \rho_j \cdot x^{j-1}
\]

The coefficients \( \lambda_j \) and \( \rho_j \) denote the fraction of edges that correspond to variable and check nodes of degree \( j \) respectively. The procedure is described as follows:

1. Initialization: Assuming that the coefficients \( \lambda_j \) and \( \rho_j \) form a vector \( w \), we chose \( L_m \) vectors randomly, with \( L_m \) the maximum variable node degree.
2. Testing: The corresponding LDPC codes are tested on the \( n_{ch} \) channel realizations, including also the noise in the system (Section III.B.C). A fixed number of iterations is set as well as a fixed number of target error probability \( P_{\text{etarget}} \). The average \( \overline{P_e} \) is derived. The value of \( E_b / N_0 \) that corresponds to \( \overline{P_e} < P_{\text{etarget}} \) is denoted for each code. The vector \( w_{i,h} \), \( i = 1, 2, \ldots, L_m \) and \( h = 1 \), that has resulted in the best \( \overline{P_e} \) for the minimum \( E_b / N_0 \) is denoted as \( z_h \).
3. New candidate degree distribution pair: New \( L_m \) vectors \( v_{i,k,h+1} \), \( i = 1, 2, \ldots, L_m \) are generated according to the following:

\[
u_{i,k,h+1} = z_h + F \cdot (w_{i,k} - w_{i,k+1})
\]

The vectors \( w_i \) and \( w_{i+1} \) correspond to randomly chosen vectors with \( i \neq i+1 \), while the parameter \( F \) controls the amplification of the variation factor, with \( F = 0.5 \). The \( k \) elements \( k = 1, 2, \ldots, O \) of the trial vectors \( v_{i,k,h+1} \) are chosen such that:

\[
v_{i,k,h+1} = \begin{cases} u_{i,k+1}, & \text{if } y \leq CR \text{ or } k = rn \\ z_{i,k+1}, & \text{if } y > CR \text{ and } k \neq rn \end{cases}
\]
optimization process to the parity matrix construction. The decoding procedure is performed in the same way as the non-optimized LDPC case. Fig. 5 shows the performance obtained when the LDPC code that corresponds to each optimized degree distribution pair is applied in the system. The solid curves “Ref1” and “Ref2” refer to the curves shown in Fig. 3a for the first and last packet length (size: 112 and size: 12) respectively. As it can be noticed from Fig. 5 and Fig. 3a the proposed designed LDPC codes generally outperform the already existing coding schemes used in the G3 standard for round 2 dB and subsequently they experience a better performance than the non-optimized LDPC codes.

Table V presents the degree distribution pairs that have come out from the procedure described in Section V.A, when impulsive noise is also considered during the optimization process. Again, the seven different code lengths are taken into account as well as the maximum variable node degrees in accordance to Table II. The correspondence of an LDPC code to an optimized degree distribution is performed in accordance to the optimization case of impulsive noise absence. We notice from the table that for a different noise scenario, the optimized code is different. This is explicable because the transmission conditions are different, thus it is anticipated that the code degree distribution pair that maximizes the performance will also differ. As a general remark from Tables IV and V, it can be concluded that for a shorter codeword length, a smaller maximum variable node degree is needed. This is reasonable, since for a shorter codeword length, the sparseness of the parity check matrix indicates the usage of smaller degrees.

Fig. 6 illustrates the performance of the optimized codes for impulsive noise scenario B, when the impulsive noise affects the data. The solid curves “Ref3” and “Ref4” refer to the curves shown in Fig. 3b for the first and last packet length (size: 112 and size: 12) correspondingly. As it can be seen from Fig. 6, the performance is improved by using the proposed code construction, in comparison to the concatenated version of convolutional plus Reed Solomon codes as well as the non-optimized case of LDPC codes. It is worth noting from Fig. 6 and Fig.3b that the performance enhancement of the proposed LDPC code version is generally in the order of 2-3 dB in comparison to the convolutional plus Reed Solomon codes for the first 6 packet lengths.

VI. CONCLUSIONS

In this paper, we have considered the construction of short LDPC codes for the NB-PLC channel. The well-established method of differential evolution for finding the degree distribution pair that enhances the performance is used. This technique is adjusted to obtain a practical approach to evaluate each potential code degree pair. For this purpose, we have extracted the candidate code performance in actual channel and noise conditions. Seven different code lengths have been examined. We tested the codes performance in the absence or presence of impulsive noise. The results show that the proposed construction of short LDPC codes gives better performance than the non-optimized QC ones. Better performance is also obtained in comparison to the baseline concatenated convolutional plus Reed Solomon codes, currently used by the G3 standard. It is also noteworthy that the optimization procedure is fundamental for achieving a performance enhancement. Thus, these codes can be considered as a candidate coding scheme for the NB-PLC channel.
Fig. 6. Bit error rate versus $E_b / N_0$ for NB-PLC, noise scenario B (presence of impulsive noise), proposed LDPC construction.

REFERENCES