Power Savings with Opportunistic Decode and Forward over In-Home PLC Networks

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Abstract—We consider a cooperative system to provide power saving, quality of service, and coverage extension over in-home power line communications (PLC) networks. We focus on a cooperative relay scheme, where the communication between source and destination nodes follows an opportunistic time division decode and forward (ODF) protocol. At the physical layer we assume the use of a multi-carrier scheme. We show that the joint problem of power and time slot allocation for the multi-carrier time division decode and forward (DF) protocol is not convex. To reduce the complexity, we propose an heuristic algorithm that considers two convex sub-problems. The validation of the algorithm is done over statistically representative in-home PLC networks. Through extensive numerical results, we show that the use of relaying allows for saving several dBs of transmitted power, yet achieving the same rate of the direct transmission. Furthermore, over multiple sub-topologies networks interconnected through circuit breakers, e.g., a multi-floor house, the relay increases the network coverage.

I. INTRODUCTION

The problem of power saving is playing an important role in future developments of communication devices. In fact, some standards have been developed to cope with this problem, e.g., the IEEE 802.3az and the HomePlug Green Phy that are respectively specified for Ethernet and power line communications (PLC). Beside the power saving problem, the designers of modern communication devices have to take into account the high communication rate that is required by multimedia applications such as HDTV or 3D virtual video games. To obtain power saving and high rates, it becomes essential to consider advanced communication techniques such as cooperative communication and cross-layer optimization. Some early work on cooperative PLC was done in [1], where it was suggested to use repeaters and distributed space time codes.

In this work we consider a cooperative system to provide power saving, quality of service, and coverage extension over in-home PLC networks. We focus on a cooperative relay scheme (see Fig. 1), where the communication between source and destination nodes follows an opportunistic decode and forward (ODF) protocol, and at the physical (PHY) layer we assume the use of multi-carrier modulation, i.e., orthogonal frequency division multiplexing (OFDM). In general, in ODF [2], the relay is used according to a time division decode and forward (DF) mode [3] whenever it allows for capacity improvements w.r.t. direct transmission (DT).

For the multi-carrier time division DF protocol we consider the joint problem of allocating the sub-channels power and time slots among the network nodes under a power spectral density (PSD) mask constraint and a rate target constraint. To the best of our knowledge, this problem has not been investigated yet. Earlier work on optimal power and time slot allocation has been done in [2], [3] for Rayleigh fading wireless relay channels. In [4], [5], the authors found the optimal power allocation for amplify and forward (AF)-OFDM relay systems assuming the destination node not reachable by the source node. In [6], the authors found the optimal power allocation for the full-duplex [3] DF-OFDM relay system. The approach followed in [4]–[6] is to pair the OFDM sub-channels between transmitter and relay to find an equivalent channel gain and thus find the optimal power allocation solving the Karush-Kuhn-Tucker (KKT) conditions [7].

We show that the joint problem of power and time slot allocation for the multi-carrier time division DF protocol is not convex. To reduce the complexity, we propose an heuristic algorithm that considers two convex sub-problems. The validation of the algorithm is done over typical in-home PLC networks that are generated using the statistical topology generator presented in [8]. Extensive numerical results show that with respect to the DT, the adoption of the proposed algorithm allows for saving several dBs of transmitted power yet achieving the same rate of DT, and further it increases the network coverage of several percentage points.

The reminder of the paper is as follows. In Section II, we summarize the network topology model presented in [8]. In Section III, we first present the ODF protocol [2], and then we focus on the power saving problem. Finally, in Section IV, we analyze the performances of the proposed power allocation algorithm. The conclusions follow in Section V.
II. NETWORK ARCHITECTURE

In this section we briefly summarize the topology model presented in [8] for single phase in-home networks. According to this model, the in-home PLC infrastructure is characterized by a wiring topology composed of two layers. As it is shown in Fig. 2, the outlets are placed at the bottom layer and are grouped and fed by the same “super node” which is referred to as derivation box. All the outlets fed by the same derivation box are nearby placed. Therefore, the location plan is divided into elements denoted as “clusters” that contain a derivation box with the associated outlets. Each cluster represents a room or a small number of nearby rooms.

Different clusters are usually interconnected through their derivation boxes with dedicated cables. This set of interconnections forms the second layer of the topology. The channels that connect pair of outlets belonging to the same cluster are called intracluster channels. Whereas, the channels associated to pairs of outlets that belong to different clusters are called intercluster channels. An intercluster channel example is shown in Fig. 2.

The main panel plays a special role inasmuch it connects the in-home network with the energy supplier network through circuit breakers (CBs). We distinguish two cases. The first case, which we refer to as single-sub-topology network, is when a single CB feeds all derivation boxes of the network. The second case, which we refer to as multi-sub-topology networks, is when many sub-topologies, each comprising a group of derivation boxes, are interconnected through CBs. The latter case can be representative of a multi-floor house.

Finally, we recall that according to the statistical topology model [8], the channel transfer function computation is done using a bottom-up method that is based on the voltage ratio approach.

In our networks we consider the communication between source and destination nodes with the help of a relay. In particular, in this paper we consider source-destination (S-D) channels defined between pair of outlets that do not belong to the same cluster, i.e., intercluster channels. As reported in [9], these channels experience higher attenuations than intracluster channels. Thus, they can have more benefits from the help of a relay. Clearly, these benefits are also dependent on the relay location. To this end, we consider for single-sub-topology networks, the following strategical configurations for the relay.

- **Random allocation derivation box (RDB).** The relay is located in a randomly selected network derivation box.
- **Backbone allocation derivation box (BDB).** The relay is located in a randomly selected derivation box that belongs to the backbone between the source and the destination nodes.
- **Source derivation box (SDB).** The relay is located in the derivation box that feeds the source node. Note that for intercluster channels the path between source and destination includes at least the derivation box that feeds the source and the one that feeds the destination.
- **Destination derivation box (DDB).** The relay is located in the derivation box that feeds the destination node.
- **Main panel single sub-topology (MPS).** The relay is located immediately after the CB of the MP.
- **Outlet relay arrangement (ORA).** The relay is located in randomly selected network outlet.

When considering multiple sub-topologies networks, we assume the source and the derivation node located in two different sub-topologies. Furthermore, we place the relay between the CBs that feed the sub-topologies, and we model each CB with an attenuation of 5 dB [10]. We refer to such configuration as main panel multiple sub-topologies (MPM).

III. OPPORTUNISTIC DECODE AND FORWARD PROTOCOL

We consider a network composed by a source (S), a relay (R) and a destination (D) node as shown in Fig. 1. Multiplexing is accomplished via TDMA, where the time is divided in frames of duration $T_f$. Each frame is divided into two time slots whose durations are $t$ and $T_f - t$. The communication between S and D follows the ODF protocol presented in [2]. In ODF the source node sends data to the destination node according to two modes, DT or DF. In DT, S transmits data to the destination occupying all the time frame $T_f$. In DF mode, S transmits its data to both R and D during the time slot $t$, then in the second slot of duration $T_f - t$, R decodes and forwards the same data to D using an independent codebook while the source is silent [3]. Finally, D decodes the message combining the data received in both time slots from both the source and the relay.

Fig. 3 shows the DT and DF modes, and the corresponding time slots allocation.

![Figure 3. DT and DF modes and corresponding time slots allocation.](image-url)
Therefore, with a frame of normalized duration $T_f = 1$, the capacity of ODF can be expressed as [2]

$$C_{ODF}(t) = \max \{ C_{DT}, C_{DF}(t) \},$$  \hspace{1cm} (1)

where $C_{DT}$ and $C_{DF}(t)$ respectively denote the capacity of DT and DF modes. They are given by

$$C_{DT} = C_{S,D}$$  

$$C_{DF}(t) = \min \{ tC_{S,R}, tC_{S,D} + (1-t)C_{R,D} \}. \hspace{1cm} (3)$$

In (2) and (3), $C_{S,D}$, $C_{S,R}$ and $C_{R,D}$ denote the capacity of the links S-D, S-R and R-D, respectively. Assuming a large number of sub-channels and additive white Gaussian noise (AWGN), they are given by [11]

$$C_{x,y} = \frac{1}{MT} \sum_{k \in \mathbb{K}_{ON}} \log_2 \left( 1 + \frac{p_{x,MODE}^k H_{x,y}^k |y|^2}{P_w^k} \right),$$

$$\{ x, y \} \in \{ \{ S, D \}; \{ S, R \}; \{ R, D \} \}, \hspace{1cm} (4)$$

where

$$SNR_{x,y}^k = \frac{p_{x,MODE}^k H_{x,y}^k}{P_w^k},$$

is the signal to noise ratio (SNR) in sub-channel $k$ for the link $x$-$y$, $T$ is the sampling period, whereas $p_{x,MODE}^k$ and $P_w^k$ denote the power transmitted by node $x$ for a given $MODE$ (DF or DT) and the noise power in sub-channel $k$. Furthermore, $\gamma_{x,y}^k$ denotes the normalized SNR for the link $x$-$y$ in sub-channel $k$.

From (1)-(3) it is interesting to note that the direct link is used whenever the capacity $C_{S,D}$ is greater than $C_{S,R}$ or $C_{R,D}$. This is true for any $t$. In the remaining cases, to see whether the communication follows the DT or the DF mode, we need to compute $C_{DT}$, $C_{DF}(t)$ and compare them as in (1) to determine the largest.

In the following, in Sub-section III-A, we discuss the capacity maximization for ODF when considering a sub-channel peak power constraint. This will be useful for the power saving treatment that follows in Sub-section III-B.

### A. Capacity Maximization

From (1), we note that the capacity of ODF is a function of both the transmitted power distribution and the time slot allocation. Therefore, in order to maximize it, when DT is used, we only need to optimally allocate the power among the sub-channels of the source node. Contrarily, when DF mode is used, we need to optimally allocate the power and the time slot between source and relay.

From now on, we assume that the network nodes have to satisfy a power spectral density (PSD) mask constraint to be compliant with the EN55022 electromagnetic compatibility (EMC) directive [12]. In such a case, it is possible to show that the sub-channels power allocation that maximizes the capacity for a point-to-point communication corresponds with the one given by the same PSD constraint [13]. Therefore, for both modes, we set $P_{x,MODE} = \overline{P}$, with $x \in \{ S, R \}$, and $k \in \mathbb{K}_{ON}$. Now, to maximize the ODF capacity (1), we only need to compute the optimal time slot duration that can be found maximizing $C_{DF}(t)$ [14], i.e.,

$$t^* = \arg\max_{t \in [0,1]} \{ C_{DF}(t) \}. \hspace{1cm} (6)$$

To solve (6), we observe that the arguments of the minimization in (3) are linear functions and thus the optimal time slot duration is given by their intersection, i.e., $t^* C_{S,R} = t^* C_{S,D} + (1-t^*) C_{R,D}$. In our previous work [10], we have extensively studied the capacity improvements given by the multi-carrier time division ODF protocol over in-home PLC networks. Therein, we have also found a global sub-optimal time slot duration and relay collocation for the capacity maximization.

### B. Power Saving

In this Sub-section we propose the use of the relay for power saving and coverage extension over PLC networks. The idea behind is the following. As discussed in the previous Sub-section, in ODF the relay is used when the DF capacity is higher than that of the DT. Now, let us suppose that the relay is used and we want to achieve a given target rate satisfying a PSD mask constraint. Then, we can have three cases. The first is when the target rate is reachable either using the DT or the DF. In such a case, since the DF capacity is higher than that of DT, the amount of power saved lowering the rate of DF to the same value. The second case is when only the DT capacity is lower than the target rate. In this case, the use of the relay can increase the network coverage. The third case is when the capacity of both modes is lower than the target rate so that the use of the relay increases the achieved rate.

To compute the power used by ODF when the communication is subjected to a target rate $R$ constraint and to a PSD constraint, we can solve the dual problem of (1), i.e.,

$$P_{ODF} = \min \{ P_{DT}, P_{DF} \},$$

where $P_{DT}$ and $P_{DF}$ respectively denote the minimum power required by the DT and the DF modes to achieve a rate $R$ under a constraint on the PSD mask. Therefore, $P_{DT}$ is the solution to the problem

$$P_{DT} = \min \sum_{k \in \mathbb{K}_{ON}} \{ P_{S,DT}^k \},$$

s.t. $C_{S,D} = R$, \hspace{1cm} (8)

$$0 \leq P_{S,DT}^k \leq \overline{P} \hspace{1cm} \forall \hspace{1cm} k \in \mathbb{K}_{ON},$$

and $P_{DF}$ is the solution to the problem

$$P_{DF} = \min \sum_{k \in \mathbb{K}_{ON}} \{ tP_{S,DF}^k + (1-t)P_{R,DF}^k \},$$

s.t. $C_{DF}(t) = \min \{ tC_{S,R}, tC_{S,D} + (1-t)C_{R,D} \} = R$, \hspace{1cm} (9)

$$0 \leq t \leq 1,$$

$$0 \leq P_{S,DF}^k \leq \overline{P},$$

$$0 \leq P_{R,DF}^k \leq \overline{P}, \hspace{1cm} \forall \hspace{1cm} k \in \mathbb{K}_{ON}.$$
Starting from (8) we note that its objective and its inequality functions are convex, but its equality constraint is not an affine function. Therefore, (8) is not a convex problem [7, pp. 136-137]. Nevertheless, we note that the equivalent problem, obtained considering the change of variables \( P_{S,DF}^{(k)} = (2^{k,DT} - 1)/\gamma_{S,D}^{(k)} \), is a convex optimization problem. Hence, the solution to (8) (assuming that it exists1) can be found imposing the KKT conditions [7, pp. 243-244], and it is given by [13]

\[
P_{S,DT}^{(k)} = P_{S,DT}^{(k)}(\nu) = \left[ \nu - 1/\gamma_{S,D}^{(k)} \right]_0^T, \tag{10}
\]

where

\[
[x]_a^b = \begin{cases}
b, & x \geq b \\
x, & a < x < b \\
a, & x \leq a,
\end{cases}
\]

and \( \nu \) is given by the solution of the second line of (8), i.e.,

\[
\sum_{k \in \mathbb{K}_{ON}} \log_2 \left( 1 + P_{S,DT}^{(k)}(\nu) \gamma_{S,D}^{(k)} \right) = MRT. \tag{12}
\]

Problem (9) is more difficult to solve than problem (8) as its objective function is not convex. This can be proved observing that the Hessian associated to its objective function is not convex. This can be shown proving that the principal minors of its Hessian are negative [7, pp. 71]. Therefore, (9) is given by the solution of the second line of (8), i.e.,

\[
\sum_{k \in \mathbb{K}_{ON}} \log_2 \left( 1 + P_{S,DF}^{(k)}(\nu) \gamma_{S,D}^{(k)} \right) = MRT. \tag{13}
\]

Once the “optimal” power distribution for the source node in DF mode has been computed, it can be used to compute the capacity of the S-D link, namely \( C_{S,D} \), in DF mode. Therefore, we can find the “optimal” power distribution for the relay node solving the following sub-problem of (9)

\[
P_{R,DF} = \min_{k \in \mathbb{K}_{ON}} \sum_{k \in \mathbb{K}_{ON}} P_{R,DF}^{(k)}(\nu) \tag{17},
\]

where \( \nu = \nu^{(k)}_{S,R} \). Therefore, we solve (7) using (10) and (19). Clearly, there are cases where a solution to the power minimization problem under a target rate and a PSD constraint does not exist for a single or for both modes. In the first case, the algorithm will choose the mode for which the solution exists. Whereas, when the solution does not exist for both modes, the algorithm will choose the mode that achieves the highest rate.

Finally, from (10), (14), (17) we see that the power allocation for the source node in both DT and DF modes, and for the relay node in DF mode follows a water-filling shape, where the maximum allowable power in each sub-channel is limited by the power constraint. Therefore, to solve \{10,12\}, \{14,15\}, and \{17,18\} we can use conventional iterative algorithms for solving power allocation problems when a sub-channel power constraint is imposed [13].

IV. Numerical Results

In order to obtain numerical results, we consider a system with \( M = 1536 \) sub-channels in the frequency band \( 0 - 37.5 \) MHz. The set \( \mathbb{K}_{ON} \) is defined according to the transmission band \( 1 - 28 \) MHz. To respect the EMC directive [12], we consider a PSD mask constraint of \( -50 \) dBm/Hz that defines the sub-channels power limit \( \mathcal{P} \) in the transmission band. Furthermore, we assume AWGN with PSD of \( -110 \) dBm/Hz and \( -140 \) dBm/Hz.
Figure 4. CCDF of capacity obtained using ODF with the relay located according to the various described configurations. The CCDF of capacity obtained assuming DT mode when no relay is connected to the network is also reported.

The network topology generator, uses the parameters in [10].

We point out that in the computation of the channel frequency response we assume modems with an input impedance of 50 Ohm for both the transmitter/receiver and the relay. Therefore, it should be noted that the presence or absence of the relay affects the frequency response between a certain pair of outlets.

Fig. 4 shows the complementary cumulative distribution functions (CCDF) of the capacity for the DT mode (2), when no relay is connected to the network (labeled as pure DT), and for the ODF protocol (1) according to the various single-sub-topology relay configurations, i.e., SDB, BDB, DDB, RDB, ORA and MPS. The time slot \( t \) is optimized as in (6). The power is transmitted at the PSD limit level, i.e., \( P^{(k)}_{\text{MODE}} = P \quad \forall \ k \in \mathcal{K}_{\text{ON}} \) and \( x \in \{ S, R \} \).

Herein, from Fig. 4, we just notice that the use of ODF gives good capacity improvements w.r.t. the DT mode. Therefore, as assumed in Sub-section III-B, it is reasonable to consider the use of the relay for power saving. This is because the use of ODF gives higher capacity values than DT, and conversely it requires less power to achieve the capacity of DT. For an exhaustive analysis of the capacity improvements given by the ODF, please refer to [10].

In order to assess the performances of the power allocation algorithm presented in Sub-section III-B, we set the target rate equal to the capacity of the S-D link when the relay is connected to the network, i.e., \( R = C_{\text{DT}} \) with \( P^{(k)}_{S,\text{DT}} = P \quad \forall \ k \in \mathcal{K}_{\text{ON}} \).

Fig. 5 shows the cumulative distribution function (CDF) of the total transmitted power for DT mode, and for ODF mode (7) when considering the various relay configurations. As we can see, for both noise levels, the best relay position is the source derivation box (SDB). We note that we could have expected this result inasmuch the SDB relay configuration also maximized the capacity. Furthermore, we note that this result agrees with what is presented in [2], [15] where it is shown that in wireless fading channels the capacity of half-duplex DF is maximized when the relay is located close to the source node. Nevertheless, it has to be said that when we consider full-duplex DF the result may change, as for instance it is shown in [16] where the best relay position is on the middle between the source and the destination node when considering OFDM over urban 3GPP wireless multipath channels.

From Fig. 5, we also notice that with probability equal to 0.8, the SDB relay configuration allows for saving more than 4 and 3 dB when the PSD noise level equals -110 and -140 dBm/Hz respectively. Notably, with the same probability, the BDB and the main panel (MPS) relay configurations allow for saving about 3 and 1.5 dB.

In Tab. I, we report the average total transmitted power (averaged over the network topologies) obtained using the proposed ODF power allocation algorithm. We also report the percentage of network topologies where the relay is used and the average transmitted power of DT. In Tab. I, \( E[\cdot] \) denotes the expectation operator. As we can see, depending on the noise power and on the relay configuration, the power saving given by the use of the relay ranges between 1.5 and 7.8 dB.

We now turn our attention to the network coverage improvements that can be obtained with the use of a relay. To this end, we consider the MPM relay configuration. This configuration can be representative of a multi-floor house, e.g., networks whose source and destination nodes are associated to outlets that belong to different floors. Since the S-D link over this kind of networks experiences high attenuation given by the presence of circuit breakers devices in the main panel, we expect that the use of the relay yields high range extension under these circumstances.

To validate our conjecture we consider two scenarios. In the first we impose a target rate of 20 Mbit/s, e.g., a typical
HDTV stream from the living room to the bed room located in a upper floor. In the second scenario we increase the target rate to 50 Mbit/s, e.g., two HDTV streams plus an Internet connection.

In Tab. II, we report the percentage of satisfied links for both scenarios considering DT and ODF with the MPM relay configuration. We also report the average total transmitted power.

We note that for the high noise level, the use of the relay increases the coverage by 24.8% and 30.4% and allows for a power saving of 3.7 and 2.7 dB for the first and the second scenario respectively. When the noise level is low, we still have high power savings using the relay but the gains associated to the coverage extension reduce below 10%. This is simply explainable observing that for low noise levels the imposed target rates are also achievable using the DT.

Clearly, the coverage extension given by the use of the relay has an equivalent in quality of service improvement. This is because the use of the relay allows a higher number of network nodes to reach a certain target rate.

Summarizing, we observe that the best relay location for single sub-topology networks is the derivation box of the source node. It should be noted that the practical realization of this layout requires to position one modem (relay) in each derivation box, and then the algorithm selects the appropriate modem for relaying. Certainly, a much simpler and feasible solution is to locate the relay in the main panel. As we have seen, this location provides high power saving when considering either single or multi-sub-topology networks, and further it yields high coverage extension for multi-sub-topology networks.

V. CONCLUSIONS

We have investigated the use of relaying for power saving and range extension over typical in-home power line communication networks. We have proposed an opportunistic decode and forward algorithm for the power and time slot allocation of multi-carrier time division relay networks. Extensive numerical results have shown that the use of the relay provides high power saving, still granting the same rate of the direct transmission. Furthermore, over multi-sub-topology networks (divided by CBs that introduce an attenuation), e.g., a multi-floor house, the use of the relay significantly increases the network coverage.

REFERENCES