Switching controllers for networked control systems with packet dropouts and delays in the sensor channel

Stefano Miani and Aurelio Carlos Morassutti
Dipartimento di Ingegneria elettrica, Gestionale e Meccanica
Università degli studi di Udine
Via delle Scienze 208, 33100 Udine, Italy
email: miani.stefano@uniud.it, aurelio.morassutti@uniud.it

Abstract. In this paper, discrete-time networked control systems (NCS) with packet dropouts and bounded-and-known delays in the sensor channel in a worst case setting are considered. Such systems are modeled as time-varying switching systems and analyzed by means of recent results in the area of switching control. By means of these results it is shown that: (a) such systems can be stabilized by a switching observer based controller if and only if a set of linear matrix inequalities (LMIs) are satisfied and (b) the satisfaction of such conditions allows to parameterize any family of stabilizing switching controllers.

Keywords: Networked control systems, switching systems, output feedback, LMI, parameterization.

1. INTRODUCTION

Systems with large-scale distributed industrial processes, where the sensors and actuators are wired through a network with limited capacity/transmission speed to a remote controller are called networked control systems (NCS). Compared to classical control systems which are each wired by point-to-point cables, these systems normally share a common transmission network to transmit the information from the sensors to the controllers or, viceversa, apply control data generated remotely to a plant. Such systems present some evident advantages, such as that of allowing easier controller/sensor reconfiguration, data monitoring and cost effective solutions but, on the other end, require ad hoc design to guarantee proper functioning under network time-varying delays and/or data drops. Such sensor-to-controller and controller-to-actuator phenomena can indeed degrade the performance of the overall control systems or even prevent it from being stabilizable. Several approaches have been proposed in the literature depending on the transmission protocol, the presence of delays and drops in the sensor and actuator channel, the delay model, and the presence of acknowledge data. We refer the reader to the excellent survey papers Zhang et al. (2001); Hespanha et al. (2007); Schenato et al. (2007).

In the setting used in the present paper it is assumed that the data available for control purposes at the $j$-th sampling time, $\hat{y}_j$ is at least one of the previous $N_{\text{max}}$ outputs, say $\hat{y}_j = y_{j-N_3}$, with $N_3 \leq N_{\text{max}}$ and that no delay in the controller-to-actuator channel is present.

In this paper, the first problem which will be investigated is the stabilization of a discrete-time networked control system with the adopted delay/dropout model using the theory on switching controllers Blanchini et al. (2009); Hespanha and Morse (2002); Lin and Antsaklis (2009), an approach which has already been used in Lin and Antsaklis (2005); Li et al. (2008); Zhang and Yu (2007) and in Yu et al. (2004) (see also Seiler and Sengupta (2001); Zhang et al. (2005); Xiao et al. (2000) for a jump system stochastic approach) for the analysis of the system performances or the synthesis via state/static output feedback controllers.

In this setting, the problem can be separated into a control and estimation problem, the latter being “the problem” for which numerous interesting contributions have appeared Schenato (2008); Sinopoli et al. (2004), which basically provide extended Kalman filters with finite or infinite gains which can be computed via ad-hoc time-varying Riccati equations.

The second problem we will focus on is that of determining proper realizations of a given family of linear time invariant (LTI) controllers $K_i(z)$, each stabilizing the discrete time plant for a constant value of the delay, so that closed loop asymptotic stability is preserved even when the system is affected by time varying measurement delays and/or dropouts.

The contribution of the present paper is that of providing a set of necessary and sufficient LMI conditions which guarantee the existence of a switching stabilizing observer-based controller and which can be solved off-line. Moreover, it is shown that by means of such conditions a solution to the second problem can always be found.

2. PROBLEM FORMULATION

We consider the problem of controlling a strictly proper $n$-dimensional discrete-time linear time invariant (LTI) plant...
\[ P = \{A, B, C\} \]
\[
x_{t+1}^i = Ax^i + Bu^t \\
y^i = Cx^i
\]
where \(x^i \in \mathbb{R}^n\), \(u^t \in \mathbb{R}^m\) and \(y^i \in \mathbb{R}^p\) and no delays or dropouts in the actuator channel, as depicted in figure 1. The system matrices can be thought of as obtained from a continuous-time plant controlled at a given sampling rate \(T_c\). The controller clock is synchronized with that of the sensor and the transmitted data is time stamped so that the sensor-to-controller delay \(\tau\) is instantly known.

Concerning the delay model, we will assume that at each sample at least one of the output data transmitted via the network reaches the controller within at most \(N_{\text{max}}\) samples and that the exact value of the plant-to-sensor delay \(\tau\) is known.

For the above system, the first problem we will focus on is the following

**Problem 1.** Determine a dynamic output feedback controller whose input is the delayed version of the plant output, \(\hat{y}^i = y^{i-N}\),
\[
\begin{align*}
q^{i+1} &= F_N q^i + G_N \hat{y}^i \\
u^t &= H_N q^i + K_N \hat{y}^i
\end{align*}
\]
(the superscript in the delay \(N = N\) has been dropped for clarity) such that the closed-loop system is asymptotically stable.

Since for the above problem to have a solution system (1) has to be stabilizable for any constant value of the delay within the interval \([0, N_{\text{max}}]\), we will work under this non restrictive assumption.

**Note 1.** Due to the adopted delay and transmission model, it might happen that \(N_{t+1} < N_t\), say the data received by the controller is not time-ordered so that, for example, the receiver might read the current value of the output \(y^i\) at time \(t\) and read the \(i\) delayed output \(y^{i+1-1}\) at time \(t+1\). This leads to the abnormal case in which the receiver might read twice the same plant output in different time instants. This is indeed quite reasonable when \(N_{t+1} \in [0, \max\{N_t+1, N_{\text{max}}\}]\), since this amounts to say that the received output is held constant for a certain number of instants unless more recent data is received (see Fig. 2, left), i.e. the received data is time ordered. It is anyway slightly more difficult to justify such phenomenon if it is exhibited without respecting the \(+1\) rule just given (see Fig. 2, right). We will dwell on this later on at the end of section 4 (comment (c) after Theorem 4).

![Figure 1. Delay System](image)

Once problem 1 has a solution, the second problem which will be considered is the following:

![Figure 2. Different delay realizations](image)
In the next sections it will be shown how, by exploiting very recent results in the area of switching systems, it is possible to provide a significant extensions of the interesting results in Yu et al. (2004).

4. DYNAMIC CONTROLLER FOR DELAYED SWITCHING SYSTEM

In the field of switching systems it is well known that asymptotic stability is equivalent to the existence of a common convex control Lyapunov function (see Lin and Antsaklis (2009)). The distinction on the class of Lyapunov functions used to test the stability leads to the following definition.

Definition 3. System (3) is asymptotically stabilizable if there exist a dynamic controller of the form (2) such that the closed loop system is asymptotically stable for any realization $N^t \in [0, N_{max}]$. System (3) is quadratically stabilizable if it is asymptotically stabilizable and the overall stability can be checked by means of a common quadratic Lyapunov function.

Though asymptotic stability is the main issue, the switching stability theory tells us that its solution passes through bilinear matrix inequalities. On the other hand quadratic stabilizability, which is a more stringent requirement, translates into tractable solutions and the existence of observer based controllers. To keep the exposition simple, we will henceforth focus our attention on the existence of quadratically stabilizing controllers only, for which the following result holds.

Theorem 4. The augmented switching dynamic system (3) is quadratically stabilizable by a dynamic linear switching controller of the form (2) if and only if there exist

(a) a matrix $\tilde{U} \in \mathbb{R}^{m \times (n+N_{max})}$ and a symmetric positive-definite matrix $\tilde{P} \in \mathbb{R}^{(n+N_{max}) \times (n+N_{max})}$, such that

\[
\begin{pmatrix}
\tilde{P} \\
\tilde{A}\tilde{P} + \tilde{B}\tilde{U}
\end{pmatrix} > 0
\]

(6)

(b) $N_{max} + 1$ matrices $\tilde{Y}_i \in \mathbb{R}^{(n+N_{max}) \times p}$ and a symmetric positive-definite matrix $\tilde{Q} \in \mathbb{R}^{(n+N_{max}) \times (n+N_{max})}$ such that

\[
\begin{pmatrix}
\tilde{Q} \\
\tilde{Q}\tilde{A} + \tilde{Y}_i\tilde{C}_i
\end{pmatrix} > 0
\]

(7)

Moreover, if the above conditions are satisfied, the controller (2) has the following observed based form

\[
\begin{align*}
\dot{x}_e^{t+1} &= (\tilde{A} + \tilde{L}_i\tilde{C}_i + \tilde{B}\tilde{J})\dot{x}_e^t - L_i y^t \\
u^t &= \tilde{J}^{\dagger}\dot{x}_e^t
\end{align*}
\]

(8)

\[
\begin{align*}
\dot{\tilde{y}}^t &= \tilde{U} \tilde{P}^{-1} \\
\tilde{L}_i &= \tilde{Q}^{-1}\tilde{Y}_i
\end{align*}
\]

(9)

Proof. See Blanchini et al. (2009)

It is worth to point out some relevant aspects of the previous result.

2 We refer the reader to Blanchini et al. (2009) for details on the general asymptotic stability case.

(a) The necessary and sufficient conditions are standard LMIs (see Boyd et al. (2004)) in the given unknowns and as such can be solved by efficient numerical tools (in the present work the software package Grant and Boyd (2008, 2009) was used). Such conditions, if satisfied, guarantee that $\tilde{A} + \tilde{B}\tilde{J}$ is asymptotically stable and that the dynamic system

\[
q^{t+1} = (\tilde{A} + \tilde{L}_i\tilde{C}_i)q^t
\]

is asymptotically stable for every evolution $i^t \in [0, N_{max}^t]$.\n
(b) The stabilizability conditions are clearly separated into a control and an estimation problem. Though this property holds in general for switching systems (say when the state update and input matrices $\tilde{A}$ and $\tilde{B}$ are also switching matrices) in this case reflects the well known fact that, for NCS affected by transmission delays or dropouts in the sensor to controller channel only, the actual stabilizability conditions are just those related to the estimation problem. Indeed, for stabilizable systems, conditions (6) can always be satisfied and the only issue remains that of estimating the state by verifying conditions (7).

(c) We have not yet been able to prove that the conditions remain necessary also when the delay model is changed. This would guarantee the existence of a quadratically stabilizing observer-based controller only when the quadratic conditions are satisfied and could be applied, for example, to check the existence of such controller when $N_{t+1} \in [0, \max\{N_{max}, N_{t} + 1\}]$ (which is basically equivalent to keep track of the most recent received data, see figure 2, left).

As a final result we provide a simple lemma which might simplify the implementation of observer based controllers and that simply states that the stabilizability conditions can be used to derive an observer based controller with a single output gain.

Lemma 5. There exists a quadratically stabilizing observer based controller of the form

\[
\begin{align*}
\dot{x}_e^{t+1} &= (\tilde{A} + \tilde{L}_i\tilde{C}_i + \tilde{B}\tilde{J})\dot{x}_e^t - L_i y^t \\
u^t &= \tilde{J}^{\dagger}\dot{x}_e^t
\end{align*}
\]

(10)

if and only if the conditions (6) and (7) in Theorem 4 are satisfied with $Y_i = Y$ for all $Y$.

In the next section we will analyze the problem of parameterizing a given family of stabilizing controllers.

5. CONTROLLER PARAMETERIZATION

Before presenting the main result, we briefly report some details on the Youla-Kucera parameterization.

Theorem 6. [Zhou et al. (1996)] Given the discrete-time MIMO plant $P(z)$

\[
P(z) : \begin{cases}
x^{t+1} = Ax^t + Bu^t \\
y^t = Cx^t
\end{cases}
\]

let $J$ and $L$ be such that $A + BJ$ and $A + LC$ are asymptotically stable. The set of all stabilizing controllers is given by the $y$-to-$u$ transfer matrix obtained from
where $T_i(z)$ is an asymptotically stable transfer matrix. Every choice of $T_i(z)$ leads to a specific stabilizing controller $R_i(z)$ which can be written as a lower LFT (see figure 3, left).

![Diagram](https://via.placeholder.com/150)

Figure 3. Lower (left) and upper (right) linear fractional transformations (LFT)

$$R_i(z) = \mathcal{F}_i(Q(z), T_i(z))$$

Moreover, for any stabilizing controller $R_i(z)$ it is possible to determine the corresponding Youla parameter $T_i(z)$ as an upper LFT (see figure 3, right)

$$T_i(z) = \mathcal{F}_u(Q^{-1}(z), R_i(z))$$

where $Q^{-1}(z)$ is the inverse transfer matrix of $Q(z)$, i.e. such that $Q(z)Q^{-1}(z) = I$.

By means of the above result it is possible to derive the state space representation of the Youla parameter corresponding to a given stabilizing controller, as per the next result.

**Theorem 7.** Given the plant $\mathcal{P} = (A, B, C)$, let $L$ and $J$ be such that $Q(z)$ as in (11) is an observer based stabilizing controller (say $A + BJ$ and $A + LC$ are stable). Then, for any stabilizing controller $R_i(z) = (F_i, G_i, H_i, K_i)$, the state representation of the Youla parameter $T_i(z)$ such that $R_i(z) = \mathcal{F}_i(Q(z), T_i(z))$ is

$$Q^{+1} = \begin{bmatrix} A + BK_iC & -BH_i \\ -G_iC & F_i \end{bmatrix} Q^t + \begin{bmatrix} BK_i - L \\ -G_i \end{bmatrix} o^t \quad v^t = \begin{bmatrix} J - K_iC \\ H_i \end{bmatrix} q^t - K_i o^t$$

(12)

**Proof.** The inverse of a square transfer function matrix with state space realization $(A_o, B_o, C_o, D_o)$ is

$$Q^{-1}(z) = \begin{bmatrix} A - B_oD_o^{-1}C_o & -B_oD_o^{-1} \\ C_o & D_o^{-1} \end{bmatrix}, \quad D_o^{-1} = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix}$$

Since for $Q(z)$ the state realization is given by

$$A_o = A + BJ + LC \quad B_o = [-L \quad B]$$

$$C_o = \begin{bmatrix} J \\ C \end{bmatrix} \quad D_o = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$$

the matrices needed for the computation of the inverse are

$$D_o^{-1} = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix}$$

$$A_o = B_oD_o^{-1}C_o = A + BJ + LC - [-L \quad B] \begin{bmatrix} J \\ C \end{bmatrix} = A$$

$$-B_oD_o^{-1} = [-B \quad -L]$$

$$D_o^{-1}C_o = \begin{bmatrix} -C \\ J \end{bmatrix}$$

and thus the state space realization of $Q^{-1}(z)$ is

$$Q^{-1}(z) = \begin{bmatrix} A, [-B \quad -L], \begin{bmatrix} -C \\ J \end{bmatrix}, \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix}$$

The upper LFT of $Q^{-1}(z)$ and $R_i(z)$ state representation is thus

$$q^t = Aq^t + Bu^t + Lo^t$$

$$v^t = -Cq^t + o^t$$

$$q^t = F_iq^t + G_iy^t$$

$$u^t = H_iq^t + K_iy^t$$

which, by proper rearrangement, results in the state representation (12).

Finally, we report the parameterization of the controllers. The proof in the general non quadratic case can be found in Blanchini et al. (2009). Here we particularize the result to the quadratic stability case and we report almost all the steps needed to derive the controllers realization.

**Theorem 8.** If the augmented system (3) is quadratically stabilizable then, for any given family of stabilizing controllers $R_i(z) = (H_i, F_i, G_i, K_i)$:

$$R_i(z) = \begin{bmatrix} q^t + 1 \\ u^t = G_iq^t + K_iy^t \end{bmatrix}$$

each stabilizing the original plant for a constant delay $0 \leq i \leq N_{max}$, then there exists nonminimal realizations of the above $K_i(z) = (H_i, F_i, G_i, K_i)$ such that the overall closed-loop system is asymptotically stable under time-varying delays.

**Proof.** Assume for simplicity that all the controllers have the same internal state dimension. Since the system is quadratically stabilizable, the observer based regulators

$$\hat{Q}_i(z) : \begin{bmatrix} \dot{x}^t + 1 \\ u^t = J\hat{x}_i^t + v^t \end{bmatrix}$$

$$V(z) = T_i(z)O(z)$$

stabilize the switching system when $T_i(z) = 0$.

Since $K_i(z)$ stabilizes the original system with a constant delay $i$ it also stabilizes the augmented system for the same constant $i$ and thus, for any $K_i(z)$ the corresponding Youla parameter $T_i(z)$

$$\hat{Q}_i(z) : \begin{bmatrix} \dot{x}^t + 1 \\ \hat{u}^t = \begin{bmatrix} A + BK_iC_i -BH_i \\ -G_iC_i \end{bmatrix} \dot{x}_i^t \quad \begin{bmatrix} BK_i - L_i \\ -G_i \end{bmatrix} o^t \end{bmatrix}$$

(13)
\[
\begin{bmatrix}
q_{t+1}^1 \\
q_{t+1}^2 \\
x_{t+1}^e
\end{bmatrix}
= 
\begin{bmatrix}
\hat{A} + \hat{B} \hat{J} + \hat{L}_i \hat{C}_i + \hat{B} \hat{D}_t \hat{C}_i \hat{B} \hat{C}_t Y_i \\
\hat{B} \hat{D}_t \hat{C}_i \\
-\hat{B} \hat{D}_t Y_i - \hat{L}_i
\end{bmatrix}
\begin{bmatrix}
q^t \\
q^t_2 \\
y^t
\end{bmatrix}
+ 
\begin{bmatrix}
J + \hat{D}_t Y_i \hat{C}_i \\
\hat{C}_i Y_i \\
-\hat{B} \hat{D}_t Y_i - \hat{L}_i
\end{bmatrix} y^t
\]

By connecting the newly realized controller to the switching plant and by setting \( e^t = q^t - x_{t+1}^e \), after some tedious algebra the closed loop system dynamics results in

\[
\begin{bmatrix}
q_{t+1}^1 \\
q_{t+1}^2 \\
x_{t+1}^e
\end{bmatrix}
= 
\begin{bmatrix}
\hat{A} + \hat{L}_i \hat{C}_i & 0 & 0 \\
\hat{B} \hat{D}_t \hat{C}_i & \hat{A}_t Y_i & 0 \\
\hat{B} (J + \hat{D}_t Y_i \hat{C}_i) & \hat{B} \hat{C}_t Y_i & \hat{A} + \hat{B} \hat{J}
\end{bmatrix}
\begin{bmatrix}
q^t \\
q^t_2 \\
y^t
\end{bmatrix}
\]

By the quadratic stabilizability conditions (see the first item after theorem 4) \( \hat{A} + \hat{B} \hat{J} \) and \( \hat{A} + \hat{L}_i \hat{C}_i \) are both asymptotically stable (for any time varying realization of \( i \)) as well as \( \hat{A}_t Y_i \), by construction. Since the system is in lower triangular form this implies that the overall system is asymptotically stable.

To conclude the present section we recap the main steps needed to (a) check the stabilizability of the delayed system and provide a set of observer based controllers and (b) reparameterize any family of constant delay stabilizing controllers \( R_i(z) = (F_i, G_i, H_i, K_i) \).

**Algorithm 1.**

1. Given the plant input matrices \( A, B, C \) and the maximum number of allowable delays/dropouts \( N_{max} \), construct the extended system matrices as in (3);
2. Solve the conditions in Theorem 4 and compute the stabilizing gains \( \hat{L}_i \) and \( \hat{J} \) in (9) to obtain a stabilizing switching observer based controller;
3. For any of the stabilizing controllers \( R_i(z) \) compute its Youla parameter by means of (13) and the transformation matrices \( V_i \) so that all the transformed autonomous systems share a common quadratic Lyapunov function;
4. Realize the controllers as in (14).

6. EXAMPLE

The data files of the examples can be found at the web page Miani and Morassutti (2009)

6.1 Example 1

We consider the stable continuous-time system

\[
\begin{align*}
P(s) &= \frac{(s^2/2 + 2/9s + 1)(s^2/2.9^2 + 2/2.9s + 1)}{(1s^2 + s)(s^2 + .002s + 1)(s^2/2.9^2 + .002s + 1)}
\end{align*}
\]

to be controlled by a 5s sampling rate digital controller in the presence of a maximum of 10 delayed or dropped measurement. Since this system passes the single output gain conditions in Lemma 5 (mind that the system is not asymptotically stable due to the presence of the pole at 0), a single output gain observer based controller could be derived.

Figures 4 and 5 depict the step response of the closed loop system under different delay realization. More precisely, the delay model “keep the most recent data” corresponding to the evolution in Fig. 4 is \( N = [1 2 3 4 1 0 1 2 3 4 5 6 6 2 3 4 5 1 2 3 4 3 4 5 6 7] \) whereas the casual delay model used in figure 5 corresponds to the delay realization \( N = [0 9 0 6 0 8 5 8 6 5 8 2 6 4 7 2 1 7 0 9 5 6 1 1 5 0 1 0 8 9 9] \).
The closed loop step response for different delay realizations is depicted in figs. 6 and 7 in the ordered and unordered data arrival case.

Figure 6. Evolution for example 2, “keep the most recent data” delay realization. \( y^t \) solid, \( \hat{y}^t \) dashed

Figure 7. Evolution for example 2, unordered sensor data. \( y^t \) solid, \( \hat{y}^t \) dashed

7. CONCLUSIONS AND RESEARCH DIRECTIONS

This work dealt with the stabilization of networked control systems affected by delays and dropouts in the sensor to controller channel only. By recasting the problem into an extended switching system and by exploiting recent results in this area a set of necessary and sufficient conditions for quadratic stabilizability of the NCS under observation was provided which allow also to reparameterize any family of “constant delay” stabilizing controllers into a family of time-varying delay stabilizing controllers.

This work is a first step towards a better understanding of the benefits deriving from the application of switching controllers to NCS and further work is definitely needed.

Amongst the open problems in this area, it is worth mentioning the possibility of deriving necessary and sufficient conditions for specific delay models, the reduction of the complexity of the observer based switching controllers, the possibility to extend the present results to systems with delays in the controller-to-actuator channel and the investigation of simpler controller reparameterizations.

REFERENCES


