Prova Teoria dei Segnali	A.A. 2008/09
laurea triennale	
Cognome e Nome ______________________________	Matricola ______________

Esercizi

1. Calcolare l'area del segnale a tempo continuo $x(t) = \mathrm{sinc}^4(2t-1)$.

2. Calcolare la trasformata di Fourier del segnale a tempo discreto

$$x(nT) = \begin{cases} \cos(2\pi f_0 nT), & n = 0, ..., N-1, \\ 0, & \text{altrimenti.} \end{cases}$$

3. Calcolare la convoluzione $z(t) = x_1 \star x_2 \star x_3(t)$ dei segnali a tempo continuo $x_1(t) = V_1 \,\mathrm{sinc}(2t/T)$, $x_2(t) = V_2 \,\mathrm{sinc}^2(t/T)$, $x_3(t) = -V_3\, \delta_R(t + 4t_0)$.

4. Calcolare la trasformata di Fourier del segnale a tempo continuo

$$x(t) = \sum_{k=-\infty}^{+\infty} \mathrm{rect}\left(\frac{t-2kT}{T}\right) - \sum_{k=-\infty}^{+\infty} \mathrm{rect}\left(\frac{t-T+2kT}{T}\right).$$

5. Calcolare la risposta impulsiva del filtro, con ingresso $a(kT)$ e uscita $b(kT)$, realizzato mediante l'equazione alle differenze

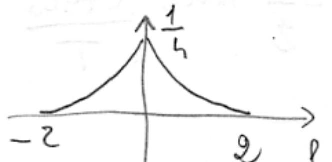
$$b(kT) = a(kT) + 0.1\, b(kT-T) - 0.01\, b(kT-2T).$$

1. $\mathrm{Area}\left(\mathrm{sinc}^4(2t-1)\right) = \mathrm{Area}\left(\mathrm{sinc}^4\, 2t\right)$

(invariante rispetto alla traslazione)

$$\int_{-\infty}^{+\infty} \mathrm{sinc}^2(2t)\,\mathrm{sinc}^2(2t)\,dt = \int_{-\infty}^{+\infty} \frac{1}{2}\left(\mathrm{triangle}\,\frac{f}{2}\right)\cdot\frac{1}{2}\left(\mathrm{triangle}\,\frac{f}{2}\right) df$$

Parseval



$$= 2\cdot\frac{1}{4}\cdot 2\,\frac{1}{3} = \frac{1}{3}$$

2. $x(nT) = s(nT)\, p(nT)$

$s(nT) = \cos 2\pi f_0\, nT$

$$p(nT) = \begin{cases} 1 & n = 0 \dots N-1 \\ 0 & \text{altrove} \end{cases}$$

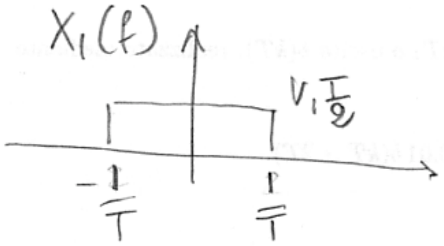
$$X(f) = \{S(f)\} * \{P(f)\}$$

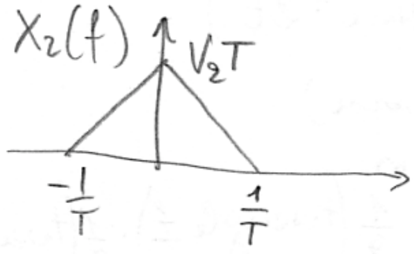
$$S(f) = \frac{1}{2}\,\delta_{\mathbb{R}/\mathbb{Z}(\frac{1}{T})}(f-f_0) + \frac{1}{2}\,\delta_{\mathbb{R}/\mathbb{Z}(\frac{1}{T})}(f+f_0)$$

$$P(f) = NT\,\mathrm{sinc}_N(fNT)\,e^{-j2\pi\frac{N-1}{2}fT}$$

$$X(f) = \frac{1}{2}NT\Big(\mathrm{sinc}_N((f-f_0)NT)\,e^{-j2\pi\frac{N-1}{2}(f-f_0)T} + \mathrm{sinc}_N((f+f_0)NT)\,e^{-j2\pi\frac{N-1}{2}(f+f_0)T}\Big)$$

3.



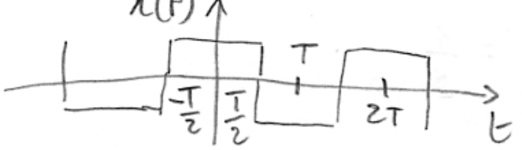


$$X_1(f)\cdot X_2(f) = \frac{V_1T}{2}X_2(f)$$

$$x_1 * x_2(t) = \frac{V_1T}{2}V_2\,\mathrm{sinc}^2\frac{t}{T}$$

$$x_1 * x_2 * x_3(t) = -V_1V_2V_3\frac{T}{2}\,\mathrm{sinc}^2\frac{t+4t_0}{T}$$

4.



$$x(t) = 2\sum_{n=-\infty}^{+\infty}\mathrm{rect}\left(\frac{t-2kT}{T}\right) - 1$$

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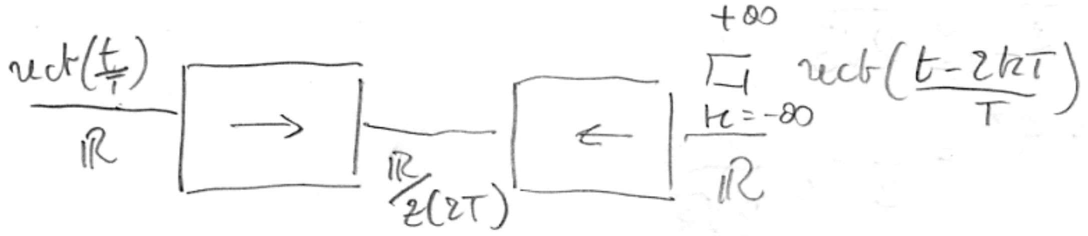
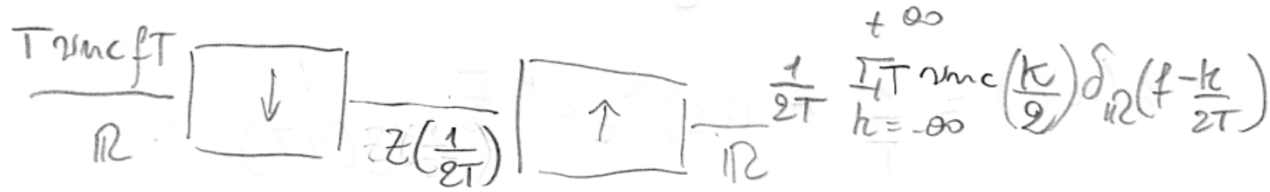
Esercizi





$$X(f) = \sum_{k=-\infty}^{+\infty} \mathrm{sinc}\,\frac{k}{2}\, \delta_{\mathbb{R}}\left(f-\frac{k}{2T}\right) - \delta_{\mathbb{R}}(f)$$

5.

$$\frac{B(f)}{A(f)} = \frac{1}{1-0{,}1\,e^{-j2\pi fT}+0{,}01\,e^{-j2\pi f2T}}$$

Poli: radici di

$$z^2 - 0.1z + 0.01$$

$$p_{12} = \frac{0.1 \pm \sqrt{0.01-4\cdot 0.01}}{2}$$

$$= 0.1\left(\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right) = 0{,}1\, e^{\pm j\frac{\pi}{3}}$$

$$H_z(z) = \frac{1}{\left(1-0{,}1\,e^{j\frac{\pi}{3}}z^{-1}\right)\left(1-0{,}1\,e^{-j\frac{\pi}{3}}z^{-1}\right)}$$

$$= \frac{A}{\left(1-0.1e^{j\frac{\pi}{3}}z^{-1}\right)} + \frac{B}{1-0.1e^{-j\frac{\pi}{3}}z^{-1}}$$

$$A = \frac{1}{2} - j\frac{1}{2\sqrt{3}}$$

$$B = A^* = \frac{1}{2} + j\frac{1}{2\sqrt{3}}$$

$$h(kT) = \frac{1}{T} A (0.1)^k e^{jk\frac{\pi}{3}} 1_0(kT)$$

$$+ \frac{1}{T} A^* (0.1)^k e^{-jk\frac{\pi}{3}} 1_0(kT)$$

$$= \frac{2}{T}(0.1)^k \,\mathrm{Re}\left(A e^{jk\frac{\pi}{3}}\right) 1_0(kT)$$

$$= \frac{2}{T}(0.1)^k \left(\frac{1}{2}\cos k\frac{\pi}{3} + \frac{1}{2\sqrt{3}}\sin k\frac{\pi}{3}\right) 1_0(kT)$$