1) $x(t) = \text{triangle}(t-1) \cdot \cos\left(2t - \frac{\pi}{3}\right)$

$$\text{triangle}(t-1) \xleftrightarrow{\mathcal{F}_1} \text{sinc}^2 f \cdot e^{-j2\pi f}$$

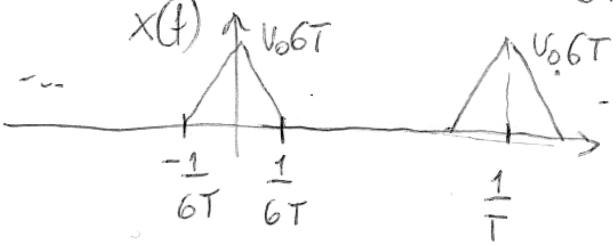
$$\cos\left(2t - \frac{\pi}{3}\right) = \frac{1}{2} e^{j2\pi\frac{1}{\pi}t} e^{-j\frac{\pi}{3}} + \frac{1}{2} e^{-j2\pi\frac{1}{\pi}t} e^{+j\frac{\pi}{3}}$$

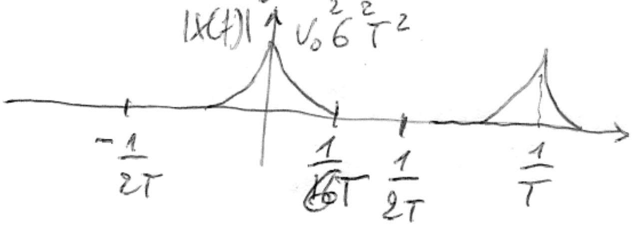
$$X(f) = \frac{1}{2} e^{-j\frac{\pi}{3}} \text{sinc}^2\left(f - \frac{1}{\pi}\right) e^{-j2\pi\left(f-\frac{1}{\pi}\right)}$$
$$+ \frac{1}{2} e^{j\frac{\pi}{3}} \text{sinc}^2\left(f + \frac{1}{\pi}\right) e^{-j2\pi\left(f+\frac{1}{\pi}\right)}$$

2) $s(kT) = V_0 \,\text{sinc}^2\left(\frac{T-kT}{6T}\right)$

$$E_s = \text{area}\left\{V_0^2 \text{sinc}^4\left(\frac{T-kT}{6T}\right)\right\} = \text{area}\left\{V_0^2 \text{sinc}^4\left(\frac{kT}{6T}\right)\right\}$$
$$= \int_{\mathbb{Z}(T)} dt \left| V_0 \text{sinc}^2\left(\frac{t}{6T}\right)\right|^2 = \int_{\mathbb{R}/\mathbb{Z}\left(\frac{1}{T}\right)} df \,|X(f)|^2$$

$$X(f) = \mathcal{F}_1\left[V_0 \,\text{sinc}^2\left(\frac{kT}{6T}\right)\right] = \text{rep}_{\frac{1}{T}}\left\{V_0 6T \,\text{triangle}(6Tf)\right\}$$





$$\int_{\mathbb{R}/\mathbb{Z}\left(\frac{1}{T}\right)} df\,|X(f)|^2 = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \left(V_0 T 6 \,\text{triangle}(6Tf)\right)^2 df = \frac{1}{6T} \cdot V_0^2 6^2 T^2 \cdot \frac{1}{3} \cdot 2$$

$$= 4 V_0^2 T = E_s$$

3)

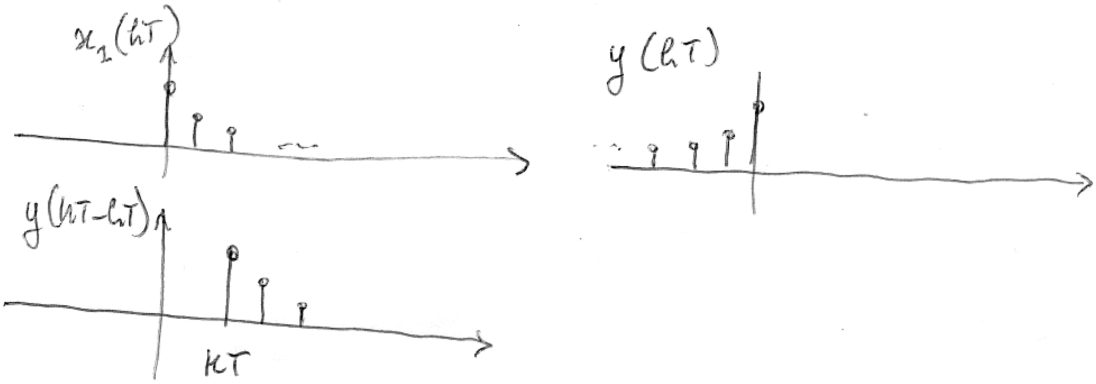
$$z(t) = x * y(t) = \left\{ e^{-\frac{2t}{T}} 1_0(t) \right\} * \left\{ e^{\frac{2t}{T}} 1_0(-t) \right\} + y(t-T)$$

$$y(t-T) = e^{2(t-T)/T} 1_0(-(t-T))$$

Calcolo di $\left\{ e^{-\frac{2t}{T}} 1_0(t) \right\} * \left\{ e^{\frac{2t}{T}} 1_0(-t) \right\} = x_1 * y(t) = z(kT)$

$$x_1(kT) = e^{-2k} 1_0(kT)$$

$$z(kT) = T \sum_{h=-\infty}^{+\infty} x_1(hT)\, y(kT-hT)$$



Per $k \geq 0$

$$z(kT) = T \cdot \sum_{h=k}^{+\infty} e^{-2h} e^{2(k-h)} = T \cdot \sum_{h=0}^{+\infty} e^{-2(k+h)} e^{-2h}$$

$$= T e^{-2k} \sum_{h=0}^{+\infty} e^{-4h} = \frac{T \cdot e^{-2k}}{1-e^{-4}}$$

Per $k < 0$

$$z(kT) = \frac{T \cdot e^{2k}}{1-e^{-4}}$$

$$z(kT) = \frac{T e^{-2|k|}}{1-e^{-4}}$$

4) $h(t) = e^{-\pi\left(\frac{t}{\sqrt{\pi}T}\right)^2}$ $\quad H(f) = \sqrt{\pi}T e^{-\pi\left(\sqrt{\pi}T f\right)^2}$

$x(t) = -\sin\left(2\pi \frac{1}{2\pi T} t\right) \qquad f_0 = \frac{1}{2\pi T}$

$y(t) = -|H(f_0)| \sin\left(\frac{t}{T} + \angle H(f_0)\right)$

$= -\sqrt{\pi}T e^{-\pi\left(\frac{\sqrt{\pi}T}{2\pi T}\right)^2} \sin\left(\frac{t}{T}\right)$

$= \sqrt{\pi}T e^{-\frac{1}{4}} \sin\left(-\frac{t}{T}\right)$

5) $x(t) = e^{-\pi\left(\frac{t}{\sqrt{\pi}}\right)^2}$

$y(t) = e^{-2(t+1)^2} = y_1(t+1)$

$y_1(t) = e^{-\pi\left(\frac{\sqrt{2}}{\sqrt{\pi}}t\right)^2}$

$z(t) = x * y(t) = x * y_1(t+1)$

$z_1(t) = x * y_1(t)$

$Z_1(f) = X(f)\, Y_1(f) = \sqrt{\pi}\, e^{-\pi\left(\sqrt{\pi} f\right)^2} \cdot \frac{\sqrt{\pi}}{\sqrt{2}} e^{-\pi\left(\frac{\sqrt{\pi}}{\sqrt{2}} f\right)^2}$

$= \frac{\pi}{\sqrt{2}} \cdot e^{-\pi\left(\sqrt{\frac{3\pi}{2}} f\right)^2}$

$z_1(t) = \frac{\pi}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{3\pi}} e^{-\pi\left(\frac{\sqrt{2}}{\sqrt{3\pi}} t\right)^2}$

$= \sqrt{\frac{\pi}{3}} \cdot e^{-\pi\left(\frac{\sqrt{2}}{\sqrt{3\pi}} t\right)^2}$

$z(t) = z_1(t+1) = \sqrt{\frac{\pi}{3}}\, e^{-\pi\left(\frac{\sqrt{2}}{\sqrt{3\pi}}(t+1)^2\right)}$