1)

$$a(kT) = (-1)^{|k|} + (0.25)^{|k|}$$

$$(-1)^{|k|} = e^{j2\pi kT\frac{1}{2T}} \longleftrightarrow \delta_{\mathbb{R}/\mathbb{Z}(\frac{1}{T})}\left(f - \frac{1}{2T}\right)$$

$$T \cdot \sum_{k=-\infty}^{+\infty} (0.25)^{|k|} e^{-j2\pi fkT} =$$

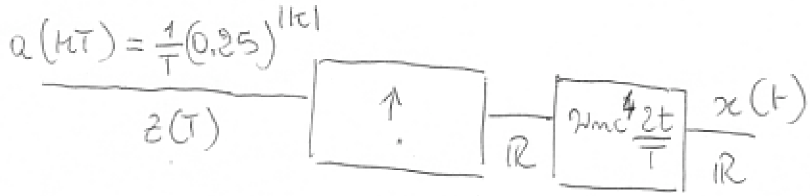
$$= T\left[\sum_{k=0}^{+\infty} 0.25^{k} e^{-j2\pi fkT} + \sum_{k=0}^{+\infty} (0.25)^{k} e^{+j2\pi fkT} - 1\right]$$

$$= \frac{T}{1-0.25e^{-j2\pi fT}} + \frac{T}{1-0.25e^{+j2\pi fT}} - T$$

$$= T\,\frac{1-(0.25)^2}{1-0.5\cos 2\pi fT + (0.25)^2}$$

$$A(f) = \delta_{\mathbb{R}/\mathbb{Z}(\frac{1}{T})}\left(f - \frac{1}{2T}\right) + \frac{T\left(1-(0.25)^2\right)}{1-0.5\cos 2\pi fT + (0.25)^2}$$

2)



$$X(f) = \underset{\substack{\uparrow \\ \text{periodica}}}{A(f)} \cdot \mathcal{F}\underset{\substack{\uparrow \\ \text{banda limitata}}}{\left[\text{sinc}^4 \frac{2t}{T}\right]} \qquad B = \frac{4}{T}.$$

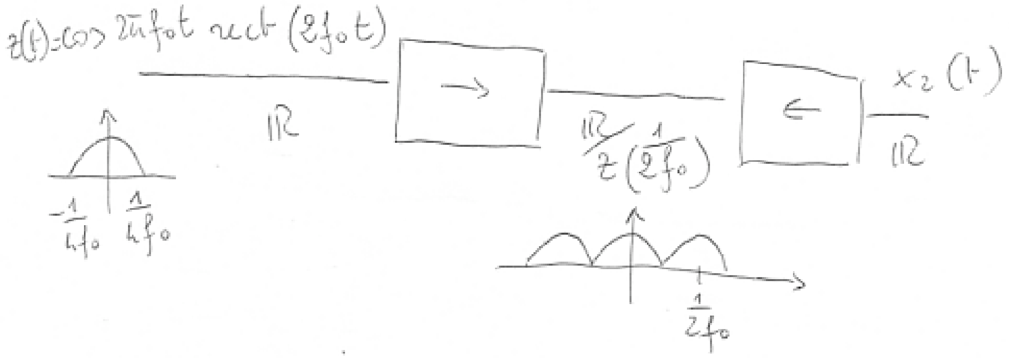
$$F_c = 2B = \frac{8}{T}$$

3) $x(t) = \left|(1+\mathrm{sinc}^2 t)\cos 2\pi f_0 t\right| = (1+\mathrm{sinc}^2 t)\,\left|\cos 2\pi f_0 t\right|$

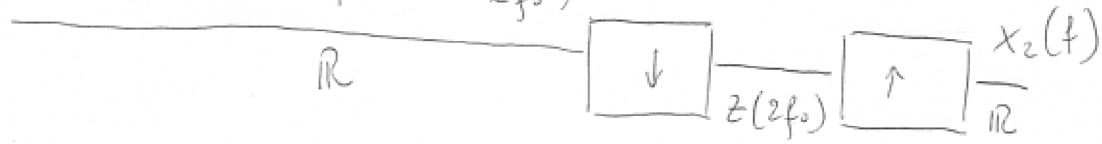
$= x_1(t)\cdot x_2(t)$

$X(f) = X_1 * X_2(f)$

$X_1(f) = \delta_{\mathbb{R}}(f) + \mathrm{triangle}(f)$

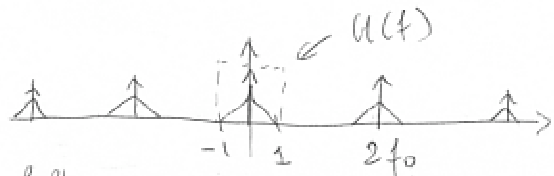


$Z(f) = \frac{1}{4f_0}\,\mathrm{sinc}\left(\frac{f-f_0}{2f_0}\right) + \frac{1}{4f_0}\,\mathrm{sinc}\left(\frac{f+f_0}{2f_0}\right)$



$$X_2(f) = 2f_0 \cdot \sum_{k=-\infty}^{+\infty} Z(2kf_0)\,\delta_{\mathbb{R}}(f-2kf_0)$$

$$X(f) = 2f_0 \sum_{k=-\infty}^{+\infty} Z(2kf_0)\,X_1(f-2kf_0)$$



Dopo il filtraggio con $H(f) = \mathrm{rect}\left(\frac{f}{2}\right)$ resta $Y(f) = 2f_0 Z(0) X_1(f)$

$y(t) = \frac{2}{\pi}\left(1+\mathrm{sinc}^2(t)\right)$

4)

$$E_x = \int_{-\infty}^{+\infty} \left| 2j\, e^{-4(t-1)^2} e^{2j} \right|^2 dt$$

$$= 4 \int_{-\infty}^{+\infty} e^{-8(t-1)^2} dt = 4 \int_{-\infty}^{+\infty} e^{-8t^2} dt$$

$$= 4 \int_{-\infty}^{+\infty} e^{-\pi \left(\frac{\sqrt{8}}{\sqrt{\pi}} t\right)^2} d = 4 \sqrt{\frac{\pi}{8}} \cdot e^{-\pi \left(f \sqrt{\frac{\pi}{8}}\right)^2} \Bigg|_{f=0}$$

$$= \sqrt{2\pi}$$

5)

$$B(f) = \frac{A(f)}{1 - 0.5\, e^{-j2\pi f 2T}}$$

si tratta di un filtro con funzione di t.f.

$$H_z(z) = \frac{1}{1 - 0.5 \cdot z^{-2}} = \frac{1/2}{1 - \frac{1}{\sqrt{2}} z^{-1}} + \frac{1/2}{1 + \frac{1}{\sqrt{2}} z^{-1}}$$

i poli $p_{12} = \pm \frac{1}{\sqrt{2}}$ sono in modulo $< 1 \Rightarrow$ il filtro è stabile in senso BIBO

$$h(kT) = \frac{1}{2T} \left(\frac{1}{\sqrt{2}}\right)^k 1_0(kT) + \frac{1}{2T} \left(-\frac{1}{\sqrt{2}}\right)^k 1_0(kT)$$

$$b(kT) = |H(f_0)| \sin\left(2\pi \frac{kT}{4T} + \angle H(f_0)\right) \; ; \quad f_0 = \frac{1}{4T}$$

$$H\left(\frac{1}{4T}\right) = \frac{2}{3}$$

$$b(kT) = \frac{2}{3} \sin 2\pi \frac{k}{4}$$