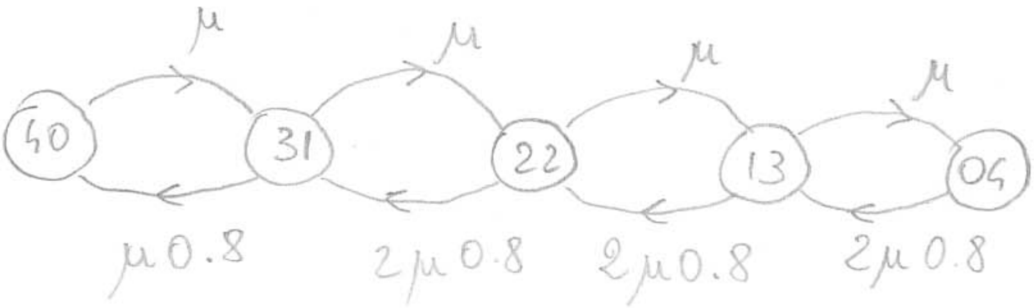
ES1

1)



$$\begin{cases} p_{40}\,\mu = \mu\, 0.8\, p_{31} \\ p_{31}\,\mu = 2\mu\, 0.8\, p_{22} \\ p_{22}\,\mu = 2\mu\, 0.8\, p_{13} \\ p_{13}\,\mu = 2\mu\, 0.8\, p_{04} \\ p_{40} + p_{31} + p_{22} + p_{13} + p_{04} = 1 \end{cases}$$

$p_{40} \simeq 0{,}261$ $\quad p_{31} \simeq 0{,}326$ $\quad p_{22} \simeq 0{,}204$

$p_{13} \simeq 0{,}128$ $\quad p_{04} \simeq 0{,}080$

Coda chiusa: esiste sempre una soluzione di equilibrio

2) $m_{s_2} = \dfrac{m_{x_2}}{\bar{\lambda}_2}$

$m_{x_2} = p_{31} + 2p_{22} + 3p_{13} + 4p_{04}$

$\dfrac{\bar{\lambda}_2}{\mu} = m_{x_2} - m_{p_2} = p_{31} + 2\left(p_{22} + p_{13} + p_{04}\right)$

3) $p(1) = p_{13}$

$$p^{(a)}(1) = \lim_{\Delta t \to 0} P\left[x_1(t) = 1 \mid A(t, t+\Delta t)\right]$$

$$= \lim_{\Delta t \to 0} \frac{P\left[A(t, t+\Delta t) \mid x_1(t) = 1\right] \cdot P\left[x_1(t) = 1\right]}{P\left[A(t, t+\Delta t)\right]}$$

$$P\left[x_1(t) = 1\right] = p(1) = p_{13}$$

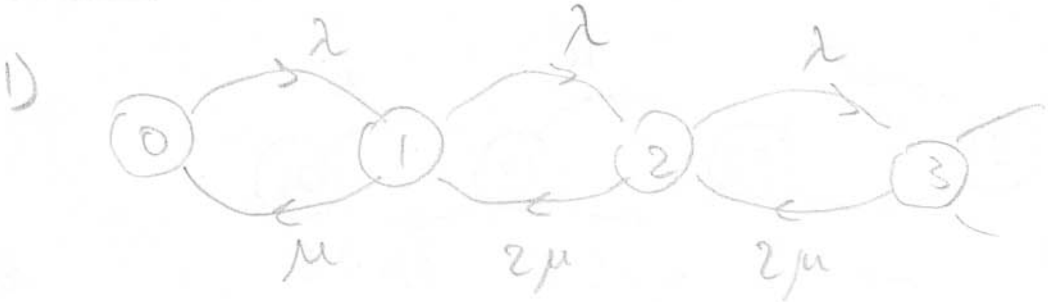
$$P\left[A(t, t+\Delta t) \mid x_1(t) = 1\right] = 2\mu\, \Delta t \cdot 0.8 + o(\Delta t)$$

$$P\left[A(t, t+\Delta t)\right] = \sum_{i=0}^{4} P\left[A(t, t+\Delta t) \mid x_1(t) = i\right] P\left[x_1(t) = i\right]$$

$$= 2\mu\, 0.8\, \Delta t\, p_{04} + 2\mu\, 0.8\, \Delta t\, p_{13} + 2\mu\, 0.8\, \Delta t\, p_{22}$$
$$+ \mu\, 0.8\, \Delta t\, p_{13} + o(\Delta t)$$

ES 2

1)



$$p_1 = \frac{\lambda}{\mu} p_0$$

$$p_2 = \frac{\lambda}{2\mu} p_1 = \frac{\lambda}{2\mu} \cdot \frac{\lambda}{\mu} p_0$$

$$p_k = \left(\frac{\lambda}{2\mu}\right)^{k-1} \cdot \frac{\lambda}{\mu} p_0$$

$\lambda = 100$

$\frac{1}{\mu} = \frac{\overline{L}}{C} = \frac{8000}{2 \cdot 10^6} =$

$\mu = 250$

$\frac{\lambda}{2\mu} < 1$

$$p_0 + \sum_{k=1}^{\infty} \left(\frac{\lambda}{2\mu}\right)^{k-1} \frac{\lambda}{\mu} p_0 = 1$$

$$p_0 = \frac{1}{1 + \frac{\lambda}{\mu} \cdot \frac{1}{1 - \frac{\lambda}{2\mu}}}$$

2) $$m_x = \sum_{k=1}^{\infty} k \cdot p_k = \sum_{k=1}^{\infty} k \left(\frac{\lambda}{2\mu}\right)^{k-1} \frac{\lambda}{\mu} p_0$$

$$= \frac{\lambda}{\mu} \cdot p_0 \cdot \frac{1}{\left(1 - \frac{\lambda}{2\mu}\right)^2}$$

$$\theta = \sum_{k=1}^{x} y_k$$

x: numero pacchetti nel sistema

y: lunghezza pacchetto

$$E[\theta] = m_x \cdot \overline{L}$$

ES 3

$$f_x(a) = 0.5\,\delta(a) + 0.25\,\beta e^{-\beta a}\,1(a) + 0.25\,\beta\, e^{\beta a}\,1(-a)$$

Se $x \in \mathcal{E}(\beta)$, $P[x \le a] = (1 - e^{-\beta a})\,1(a)$.

$y = -x$ ha distribuzione

$$P[y \le a] = P[x \ge -a] = \begin{cases} e^{\beta a} & a \le 0 \\ 1 & a > 0 \end{cases}$$

e densità

$$f_y(a) = \beta e^{\beta a}\,1(-a)$$

Quindi $f_x(a)$ è la combinazione convessa di 3 densità di probabilità

$$\delta(a) \longleftrightarrow P[x_1 = 0] = 1$$
$$\beta e^{-\beta a}\,1(a) \longleftrightarrow x_2 \in \mathcal{E}(\beta)$$
$$\beta e^{\beta a}\,1(-a) \longleftrightarrow -x_3 \in \mathcal{E}(\beta)$$

```
function y = genere()
u = rand;
if u ≤ 0.5
    y = 0;
else if u ≤ 0.75
    u2 = rand;
    y = -1/β ln u2
  else
    u3 = rand
    y = 1/β ln u3
  end
end
```