Prova Teoria dei Segnali laurea triennale	A.A. 2007/08
Cognome e Nome ______________________	Matricola ____________

Esercizi

$$1)\ E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} 25\, t^2 e^{-\frac{t^2}{2}} dt$$

$$= 25 \int_{-\infty}^{+\infty} t \quad t \cdot e^{-\frac{t^2}{2}} dt = (\text{per parti})$$

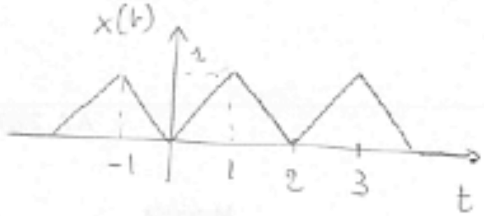
$$= 25 \left[-e^{-\frac{t^2}{2}} \cdot t \Big|_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}} dt \right] = 25 \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}} dt$$

$$= 25 \sqrt{2\pi}$$

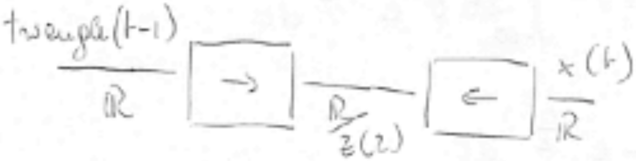
$$\int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}} dt = \int_{-\infty}^{+\infty} e^{-\pi\left(\frac{t}{\sqrt{2\pi}}\right)^2} dt = \sqrt{2\pi}\, e^{-\pi\left(\sqrt{2\pi} f\right)^2} \Big|_{f=0}$$

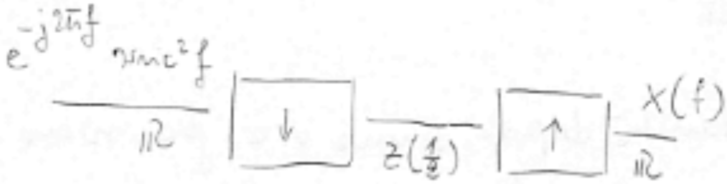
$$= \sqrt{2\pi}$$

2)



$$x(t) = \text{rep}_2 \ \text{triangle}(t-1), \quad t \in \mathbb{R}$$





$$X(f) = \frac{1}{2} \sum_{k=-\infty}^{+\infty} e^{-j2\pi \frac{k}{2}} \text{sinc}^2\left(\frac{k}{2}\right) \delta_{\mathbb{R}}\left(f - \frac{k}{2}\right)$$

$$= \frac{1}{2} \sum_{k=-\infty}^{+\infty} (-1)^k \text{sinc}^2\left(\frac{k}{2}\right) \delta_{\mathbb{R}}\left(f - \frac{k}{2}\right)$$

3) La tf di Fourier di $\text{rect}\left(\frac{kT}{5T}\right)$ è $5T \,\text{sinc}_5(f5T)$.
Dunque, $H(f) = \mathcal{F}\left[e^{j2\pi kT\frac{1}{2T}} \text{rect}\left(\frac{kT}{5T}\right)\right] = 5T\,\text{sinc}_5\left(5T\left(f - \frac{1}{2T}\right)\right)$
Dato che gli esponenziali complessi sono autofunzioni dei filtri, l'uscita $y(kT)$ all'ingresso $x(kT) = e^{+j2\pi kT\frac{1}{2T}}$ risulta

$$y(kT) = x(kT) \cdot H\left(\frac{1}{2T}\right) = 5T\,(-1)^k$$

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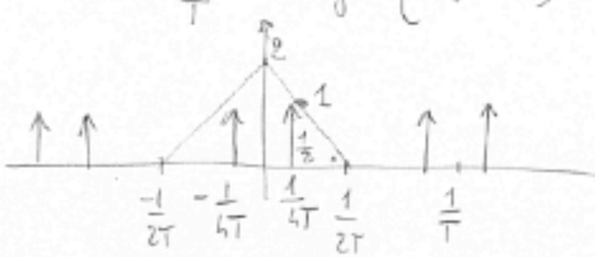
4) $$\xrightarrow[Z(T)]{x(nT)} \boxed{\uparrow} \xrightarrow[\mathbb{R}]{} \boxed{h(t)} \xrightarrow[\mathbb{R}]{y(t)}$$

$$\xrightarrow[\mathbb{R}/Z(\frac{1}{T})]{X(f)} \boxed{\leftarrow} \xrightarrow[\mathbb{R}]{} \boxed{H(f)} \xrightarrow[\mathbb{R}]{Y(f)}$$

$$x(nT) = \cos 2\pi\, nT\, f_0 \qquad f_0 = \frac{1}{4T}$$

$$X(f) = \frac{\delta_{\mathbb{R}/Z(\frac{1}{T})}(f-f_0) + \delta_{\mathbb{R}/Z(\frac{1}{T})}(f+f_0)}{2}$$

$$H(f) = \frac{2T}{T}\ \text{triangle}\left(f \cdot 2T\right)$$



$$Y(f) = \frac{\delta_{\mathbb{R}}\left(f-\frac{1}{4T}\right) + \delta_{\mathbb{R}}\left(f+\frac{1}{4T}\right)}{2}$$

$$y(t) = \cos 2\pi \frac{1}{4T} \cdot t \qquad t \in \mathbb{R}$$

5) $H(f) = \dfrac{T}{\left(1-0.1e^{-j2\pi fT}\right)\left(1-0.3e^{-j2\pi fT}\right)}$

$$= \frac{-\frac{1}{2}T}{1-0.1e^{-j2\pi fT}} + \frac{\frac{3}{2}T}{\left(1-0.3e^{-j2\pi fT}\right)}$$

$$h(kT) = -\frac{1}{2}(0.1)^{k}1_0(kT) + \frac{3}{2}(0.3)^{k}1_0(kT)$$

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Esercizi

1) $E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} 25\, t^2 e^{-\frac{t^2}{2}} dt$

$= 25\sqrt{2\pi} \int_{-\infty}^{+\infty} t^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$

$\frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$ è la densità di prob. di una v.a. gaussiana a media nulla e varianza $\sigma^2 = 1$. Dunque

$$\int_{-\infty}^{+\infty} t^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = \sigma^2 = 1$$

$E_x = 25\sqrt{2\pi}$

In alternativa l'esercizio si risolve, posto

$$s(t) = 25 t^2 e^{-\frac{t^2}{2}}$$

calcolando

$$\int_{-\infty}^{+\infty} s(t) dt = S(0)$$

La tf. di Fourier di $s(t)$ si calcola a partire da quella di $y(t) = e^{-\frac{t^2}{2}}$ e sfruttando due volte la regola

$$-j2\pi t\, y(t) \longleftrightarrow \frac{dY(f)}{df}$$